Risk-Adjusted Portfolio Optimization: Monte Carlo Simulation and Rebalancing

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Abstract

This study evaluates the risk-adjusted performance of a diversified portfolio in the Indian financial market from 2011 to 2021, incorporating Nifty 50 stocks and new-age assets. Leveraging Monte Carlo simulations and mathematical optimization, the research identifies an optimal portfolio on the efficient frontier. Integration of the Black-Litterman model provides a comparative analysis, emphasizing the impact of investor views. Despite transaction costs, optimized portfolios outperform the Nifty 50 index, with the rebalanced portfolio demonstrating higher cumulative returns. Key findings include TCS. NS is a leader in share price, HDFCBANK.NS showcasing stability and alternative assets exhibit higher volatility but have the potential for amplified returns. This research offers valuable insights for investors seeking resilient strategies in the Indian financial landscape.

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Keywords: Portfolio Optimization; Nifty 50; Monte Carlo Simulation; Transaction Costs; Rebalancing Strategies

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INTRODUCTION

Background and Context of the Study

Portfolio optimization is a crucial aspect of investment management, aimed at achieving the optimal allocation of assets to maximize returns while minimizing risk. As noted by Markowitz (1952), the founder of Modern Portfolio Theory (MPT), diversification plays a vital role in reducing portfolio risk. According to Markowitz's seminal work, "Portfolio Selection," an investor can improve risk-adjusted returns by combining assets with varying levels of risk and return. The concept of portfolio optimization has evolved significantly since Markowitz's groundbreaking work. Several researchers have extended his ideas and developed innovative approaches to enhance investment decision-making. For instance, Sharpe (1966) introduced the widely-used Sharpe ratio, which measures the excess return earned by an investment per unit of risk taken. This metric has been instrumental in evaluating the performance of portfolio optimization strategies.

In recent years, advancements in computational power and access to extensive financial data have opened up new avenues for portfolio optimization research. Researchers have explored sophisticated techniques, including the Black-Litterman model proposed by Black and Litterman (1992), which incorporates investors' views on expected returns and asset allocation. Moreover, risk parity strategies, as introduced by Arnott et al. (2012), have gained traction as an alternative to traditional mean-variance optimization. These strategies allocate assets based on their risk contribution to the portfolio, aiming to achieve a more balanced risk exposure.

Despite the abundance of research on portfolio optimization, challenges and debates persist. Researchers continue to explore alternative models, refine existing methodologies, and investigate new risk measures to improve portfolio performance. The optimization process involves making assumptions, such as the stability of correlations and expected returns, which can introduce uncertainty into the results. Additionally, the impact of transaction costs, liquidity constraints, and market frictions remains an active area of research.

The stock market has always held a certain allure, drawing in investors and financial analysts alike with the promise of opportunities and the challenge of managing risk. In the context of India, the Nifty 50 index has emerged as a prominent and closely-watched benchmark. Comprising the 50 largest and most liquid stocks traded on the National Stock Exchange (NSE), the Nifty 50 serves as a vital barometer for the Indian equity market. Investors, traders, and financial institutions rely on it to gauge the market's pulse.

Research Objectives and Significance

The study aims to evaluate the risk-adjusted performance of a diversified portfolio comprising five stocks from different sectors of the Nifty 50 index and additional new-age assets. This study will provide valuable insights for investors seeking to diversify their portfolios beyond traditional equities and explore the potential of alternative assets within the Indian context. Leveraging analytical tools like Monte Carlo simulations and efficient frontier plotting, it unravels the complexities of the Indian stock market. The study's findings offer insights for investors to enhance returns and mitigate risk in their investments.

Overview of Nifty 50 Stocks and New Age Assets Selected for Optimization:

The portfolio under consideration for optimization comprises a combination of traditional Nifty 50 stocks and new-age assets, reflecting a diversified investment strategy that encompasses various sectors and emerging asset classes. The selected assets are chosen for their potential to contribute to an optimal balance between returns and risk.

The inclusion of these new-age assets alongside traditional Nifty 50 stocks aims to create a well-rounded portfolio, considering the evolving dynamics of financial markets. The optimization strategy will explore the potential synergies and risk mitigation benefits arising from the combination of these diverse assets. Through analytical tools and methodologies such as Monte Carlo simulations and efficient frontier plotting, the study seeks to provide insights for investors seeking an optimal balance in their investment portfolios, harnessing the strengths of both traditional and emerging asset classes.

LITERATURE REVIEW

Portfolio optimization and rebalancing are foundational strategies for investors seeking to strike the delicate balance between maximizing returns and managing risks. While these concepts have deep-rooted origins, contemporary research has ushered in innovative methodologies to refine and enhance these practices.

Markowitz (1952) introduced the power of Monte Carlo simulations in the pursuit of optimal portfolios. This flexible approach provides investors with a robust tool capable of accommodating a multitude of assets, offering a dynamic means of portfolio optimization. Fama (1970) significantly contributed to the efficient market hypothesis, a cornerstone of the efficient frontier. His research, rooted in the notion that asset prices incorporate all available information, challenged investors to achieve returns surpassing the market average consistently. Bernstein (2000) expanded upon this foundation by delving into intelligent asset allocation techniques. Emphasizing the need for portfolios that strike the ideal balance between returns and risk, Bernstein's work continues to guide investors in their pursuit of optimal allocation strategies. Detemple (2000) introduced the concept of Monte Carlo simulations to determine optimal portfolios. This flexible approach can handle many assets, offering a dynamic means of portfolio optimization. Coleman, Li and Patron (2007) explore total risk minimization in incomplete markets, focusing on approximating option payoffs. They employ a piecewise linear criterion and propose a method using Monte Carlo simulation and spline approximations to compute optimal hedging strategies, comparing them to traditional quadratic and shortfall risk minimizing strategies. Results suggest that piecewise linear risk minimization may lead to lower hedging costs and different, potentially improved, hedging strategies, with a tendency to under-hedge options. Guastaroba (2008) built upon Markowitz's mean-variance model to investigate multi-period portfolio rebalancing. Their optimization model, rooted in the Conditional Value at Risk, illuminated the potential benefits of rebalancing in capitalizing on new information. Hau (2008) ventured into the realm of international portfolio rebalancing, analyzing data from an extensive sample of 6,500 funds. Their research uncovered compelling evidence of fund managers utilizing rebalancing to stabilize risk exposures, shedding light on essential considerations for models featuring financial intermediaries. Yu (2011) brought forth a diverse array of portfolio rebalancing models, each gauged against criteria encompassing risk, return, short selling, skewness, and kurtosis. Their findings challenged the conventional 'buy and hold' strategy, showcasing alternative approaches for optimizing returns. Woodside-Oriakhi (2013) navigated the complexities of portfolio rebalancing within the realm of transaction costs and varying investment horizons. Their insights underscored the pivotal role played by the investment horizon in decision-making, especially when considering associated costs. Elton et al. (2014) offered a comprehensive overview of modern portfolio theory and investment analysis, emphasizing the construction of efficient frontiers. Their work highlighted the critical consideration of risk-adjusted returns in portfolio optimization, acknowledging that raw returns alone do not paint the full picture. Ekren (2015) shifted the spotlight to portfolio rebalancing, a critical component of effective asset allocation. By devising formulas to calculate optimal rebalancing frequencies, the study illuminated the cost-saving potential of less frequent rebalancing, reshaping conventional wisdom. Almahdi and Yang, (2017) present a study on optimal asset allocation using recurrent reinforcement learning (RRL) and a coherent risk measure, expected maximum drawdown (E(MDD)). They introduce a method that incorporates the Calmar ratio to determine buy and sell signals and asset allocation weights. Results indicate that this approach outperforms other RRL-based strategies, such as the Sharpe ratio and Sterling ratio. Additionally, variable-weight RRL portfolios excel over equal-weight RRL portfolios, even in various transaction cost scenarios. The study proposes an adaptive E(MDD) risk-based RRL portfolio rebalancing system, showing responsiveness to transaction costs and consistent outperformance of hedge fund benchmarks. Liu (2018) advanced the discourse by providing analytical solutions for optimal portfolio rebalancing across multiple risky assets. By delineating the no-trade region and optimal portfolios under various correlation scenarios, Liu's work offered valuable insights into the art of portfolio rebalancing. Babazadeh and Esfahanipour (2019) present a novel portfolio optimization model that addresses the fat tail characteristic and extreme events in asset returns. This model utilizes Value at Risk (VaR) with Extreme Value Theory (EVT) for improved risk assessment and incorporates real trading constraints. To solve the resulting non-convex NP-hard problem efficiently, a modified Non-dominated Sorting Genetic Algorithm (NSGA-II) is introduced. Comparative analysis against other VaR estimation methods and benchmark algorithms using S&P 100 indices data demonstrates the effectiveness of the proposed NSGA-II in mean-VaR portfolio optimization, particularly in the low-risk domain of the Pareto front, outperforming other algorithms. Bavarsad Salehpoor and Molla-Alizadeh-Zavardehi (2019) introduce a novel portfolio optimization approach using hybrid meta-heuristic algorithms for various risk measures, including mean absolute deviation (MAD), semi-variance (SV), and variance with skewness (VWS). Algorithms like Electromagnetism-like algorithm (EM), particle swarm optimization (PSO), genetic algorithm (GA), genetic network programming (GNP), and simulated annealing (SA) are employed. A diversification mechanism enhances diversity and mitigates local optimization. The model is validated with 50 Iranian stock exchange factories, demonstrating its effectiveness through comprehensive performance metrics and analysis of variance technique. Gnabo and Soudant (2022) investigate European equity mutual funds' portfolio rebalancing in response to conventional and unconventional monetary policies. Using data from 2002 to 2016, they find that these funds tend to reallocate assets towards mid-cap and core stocks in developing economies following unconventional policies, moving away from small-cap and value stocks in developed countries. Managers also prioritize their preferred investment strategies. The study suggests that managers favour safer and more familiar stocks after unconventional policy announcements, potentially reducing information asymmetry. The study also shows that factors like fund size, returns volatility, and expense ratio influence the strength of rebalancing. (Lim, Cao and Quek, 2022) introduces practical portfolio models that incorporate investor sentiments for optimizing asset allocations. These models use objectives like Omega ratio and Conditional Value-at-Risk (CVaR) while accounting for transaction costs, short selling, and lower weight bounds. Sentiment analysis from Twitter data enhances dynamic portfolio rebalancing. Empirical results, using S&P 500 data, demonstrate that sentiment-triggered dynamic rebalancing portfolios outperform fixed-period rebalancing models and naive diversification portfolios. This flexibility in asset allocation based on sentiment leads to improved portfolio performance and effective asset management. (da Costa, Pesenti and Targino, 2023) introduce a numerical framework for risk budgeting portfolios that relies solely on return simulations. Their approach includes a cutting planes algorithm for determining portfolio weights based on various risk measures. They specifically offer versions for Expected Shortfall and utilize Stochastic Gradient Descent (SGD). Comparing their method to traditional convex optimization solvers, they apply it to real financial data, showcasing the effectiveness of their approach in constructing risk budgeting portfolios. (Chelikani, Marks and Nam, 2023) find that the volatility feedback effect significantly influences the intertemporal risk-return tradeoff. It strengthens the risk-return relationship with adverse market news but weakens it with positive news, potentially leading to market crashes even without macroeconomic uncertainties. The asymmetric effect results from a negative correlation between concurrent volatility and price changes, evident across various market conditions, including sentiment levels and business cycles.

In the quest for maximizing returns while managing risks, the literature reveals the dynamic interplay of innovative methodologies and established theories. The incorporation of Monte Carlo simulations, optimization models, and analytical solutions empowers investors to navigate the complexities of portfolio optimization and rebalancing. Furthermore, the strategic significance of rebalancing in capitalizing on new information underscores the need for a nuanced approach, considering factors such as investment horizon and rebalancing frequency. International fund managers' proactive rebalancing practices add a crucial layer to macroeconomic models, enriching our understanding of portfolio dynamics in a global context.

RESEARCH METHODOLOGY AND DATA COLLECTION

Data Collection

Five equity sample stocks were exclusively drawn from the Nifty 50 index, representing a diversified collection of India's leading blue-chip stocks across various sectors. However, recognizing the evolving landscape of financial markets and the potential benefits of diversification, the sample has been expanded to include a combination of Nifty 50 stocks and new-age assets. The portfolio comprises stocks from sectors like energy, banking, IT, consumer goods, and infrastructure, alongside newer asset classes like Cryptocurrency (Bitcoin), Gold ETF, and a Mutual fund focused on natural resources and new energy, buying equity in the energy sector comprises crude oil only which can be dealt in the derivative market.

To initiate the journey of analyzing the asset portfolio, this study selects Reliance Industries (RELIANCE.NS), HDFC Bank (HDFCBANK.NS), Tata Consultancy Services (TCS.NS), Hindustan Unilever (HINDUNILVR.NS), and Larsen and Toubro (LT.NS), Bitcoin (BTC-INR), SBI Gold ETF (SETFGOLD.NS), DSP Natural Resource & New Energy Fund (0P0001BA05.BO).

To gather the requisite historical stock price data, this study relied on Yahoo Finance as our primary data source. Yahoo Finance is renowned for its accessibility and reliability in providing comprehensive financial data. Daily adjusted closing prices have been collected for these Nifty 50 stocks and new age assets over a substantial timeframe, spanning from January 1, 2011, to December 31, 2021 i.e. 10 years. This extensive dataset forms the bedrock of our analysis, allowing us to draw valuable insights and make informed decisions.

Here is a Python code snippet illustrating how the historical stock data have been downloaded using the yfinance library:

```
# Define the list of diverse assets
import yfinance as yf
stocks = ['RELIANCE.NS', 'TCS.NS', 'HDFCBANK.NS', 'HINDUNILVR.NS',
'LT.NS', 'BTC-INR', 'SETFGOLD.NS',  '0P0001BA05.BO']
# Download historical stock data using Yahoo Finance
def download_data(stocks, start_date, end_date):
    data = yf.download(stocks, start=start_date,
end=end_date)['Adj Close']
    return data
# Set start and end dates for data
start_date = '2011-01-01'
end_date = '2021-12-31'
# Download data
stock_data = download_data(stocks, start_date, end_date)
```

This code allowed us to acquire a comprehensive dataset for our analysis for the above mentioned period.

Methodology

Descriptive statistics

Our initial step involves computing descriptive statistics for the collected data. This study starts by calculating the daily returns for each stock using the formula:

$$\text{Daily Return} = \frac{\text{Adjusted Close Price}_t}{\text{Adjusted Close Price}_{t-1}} - 1$$

Where 't' represents the trading day. This step enables us to gauge the historical performance of individual stocks and their associated risk.

Mean daily return

To construct efficient portfolios, mean daily returns have been calculated for each stock using the formula:

$$\text{Mean Daily Return} = \frac{1}{N}\sum_{i=1}^{N}\text{Daily Return}_i$$

Where '*N*' represents the number of trading days in the data.

Portfolio optimization

Our research extends to optimizing portfolio construction. This study employs the Monte Carlo simulation technique to explore a vast array of possible portfolio weightings. For each simulation, we randomly allocate weights to each stock, ensuring that they sum up to 1.0. The expected portfolio return (*Rp*) and volatility (*σp*) are computed as follows:

$$R_p = \sum_{i=1}^{N}\text{Weight}_i \cdot \text{Mean Daily Return}_i \cdot 252$$

$$\sigma_p = \sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N}\text{Weight}_i \cdot \text{Weight}_j \cdot \text{Cov}_{ij} \cdot 252}$$

Where N is the number of stocks, 252 represents the number of trading days in a year, and Weight_i is the weight of stock i in the portfolio.

The Sharpe Ratio (*SR*) is then calculated as:

$$SR = \frac{R_p}{\sigma_p}$$

This ratio helps us identify portfolios that offer the best risk-adjusted returns.

Monte Carlo Simulation and Efficient Frontier

- It generates a specified number of random portfolios (num_ports) with random weights assigned to the stocks in nifty50_stocks. Each portfolio's return, volatility, and Sharpe ratio are calculated based on the mean daily return (mean_daily_ret) and covariance matrix (cov_matrix) of the stocks.
- Mathematical Optimization: The get_ret_vol_sr function calculates the return, volatility, and Sharpe ratio for a given set of weights. The neg_sharpe function returns the negative Sharpe ratio for optimization purposes (minimizing the negative is equivalent to maximizing the positive). The minimize function from the SciPy library is used to find the portfolio weights that maximize the Sharpe ratio. Constraints ensure that the sum of weights is 1. The optimized weights are stored in optimal_weights_math.
- Efficient Frontier: The efficient frontier represents a set of optimal portfolios that offer the highest expected return for a defined level of risk. It calculates the volatility for various expected returns along the efficient frontier using the minimize_volatility function and the minimize function. The results are stored in frontier_volatility. The portfolio with the maximum Sharpe ratio is identified, and its weights are stored in optimal_weights.
- Plotting: A scatter plot is created with volatility on the x-axis, return on the y-axis, and colour-coded by the Sharpe ratio. The portfolio with the maximum Sharpe ratio is highlighted in red. The efficient frontier is plotted as a green dashed line. The resulting plot provides a visualization of the portfolios generated through the Monte Carlo simulation, the optimized portfolio from mathematical optimization, and the efficient frontier.

Black-Litterman Model and Efficient Frontier Comparision

- The black-litterman function calculates the expected returns based on the Black-Litterman model. It takes as inputs the expected market returns (expected_market_returns), the blending factor (tau), the matrix of views (P), the vector of views (Q), the uncertainty matrix of views (omega), and the covariance matrix of asset returns (cov_matrix). The equilibrium excess returns (pi) are calculated based on the blending factor and the covariance matrix. Adjustments to the expected returns are made based on views and uncertainties to obtain the Black-Litterman expected returns (expected_returns_bl).
- Inputs for Black-Litterman: The risk aversion parameter (risk_aversion) is set to 2.0 (you can adjust this based on investor risk aversion). The blending factor (tau_bl) is set to 0.025 (you can adjust this for the Black-Litterman model). The matrix of views (P) is set as an identity matrix, indicating no strong views. The vector of views (Q) is set to zero, indicating market equilibrium. The uncertainty matrix of views (omega) is calculated as the product of the blending factor (tau_bl) and the covariance matrix of asset returns (cov_matrix.values).
- Applying the Black-Litterman Model: The expected market returns are calculated based on historical data using mean_historical_return from the expected_returns module. The returns are adjusted for daily data by dividing by 252. The Black-Litterman model is applied using the

defined inputs, and the resulting expected returns are used in the PyPortfolioOpt EfficientFrontier class.

- Efficient Frontier Comparison: The efficient frontier is calculated for both the Monte Carlo simulation and the Black-Litterman model. The code generates random portfolios using Monte Carlo simulation and calculates their returns, volatilities, and Sharpe ratios. The efficient frontiers obtained through Monte Carlo simulation and the Black-Litterman model are plotted on the same graph for comparison. The plot includes a colour bar indicating the Sharpe ratio for each portfolio.

Overall, the code provides a visual comparison of the efficient frontiers obtained through traditional Monte Carlo simulation and the Black-Litterman model, demonstrating how incorporating views and uncertainties can impact portfolio optimization.

Portfolio rebalancing

Effective portfolio management involves periodic rebalancing to maintain desired asset allocations and risk-return profiles. Our methodology incorporates a rebalancing criterion that examines the portfolio's performance every 90 days, equivalent to each quarter. If the portfolio's asset weights deviate significantly from the target allocation, rebalancing is initiated to restore the intended balance. When the rebalancing criterion is met, this study systematically adjusts the portfolio's asset weights to return them to their target values. This process may involve buying or selling assets within the portfolio. By simulating this rebalancing activity, we evaluate its impact on portfolio performance over time. The quarterly rebalancing schedule is a practical approach, enabling investors to make timely adjustments to their portfolios as market conditions evolve.

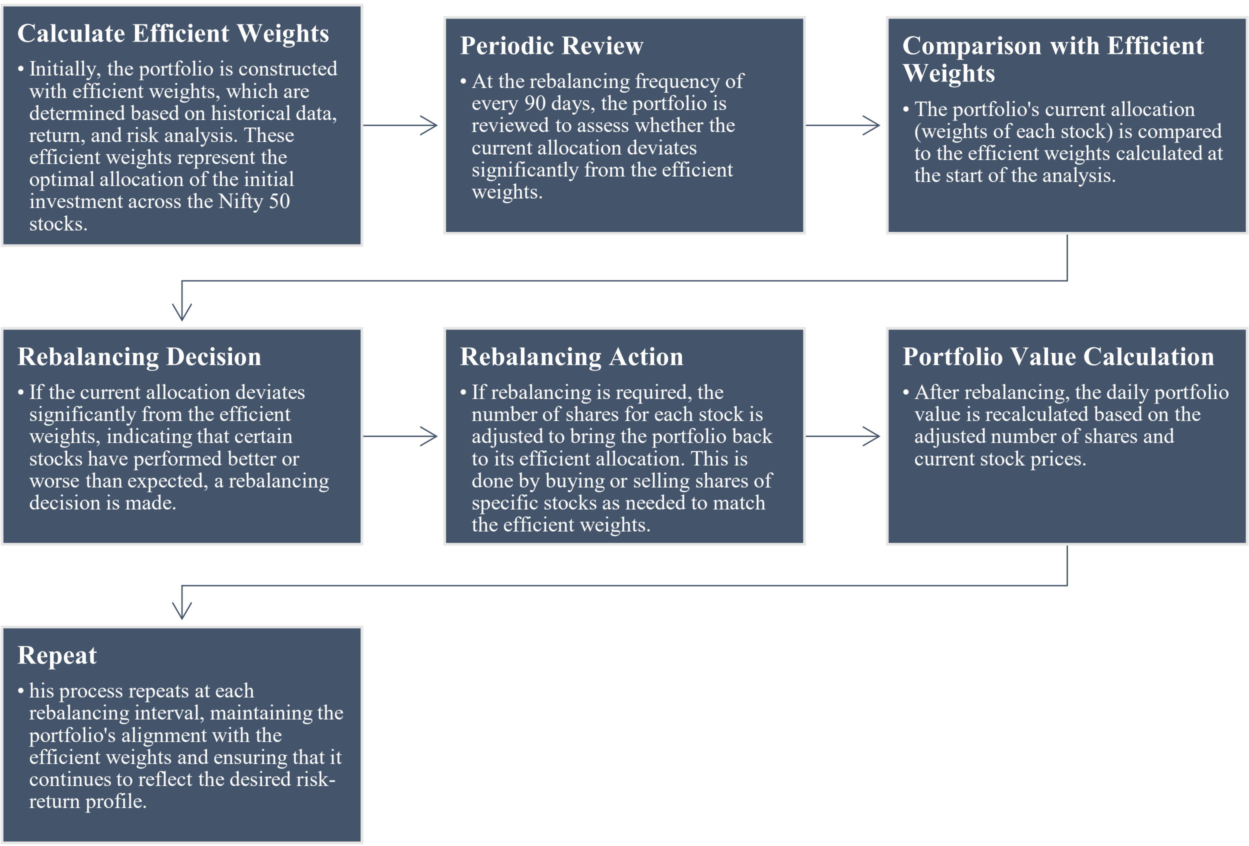

Fig1: Portfolio Rebalancing Criteria

Transaction Costs and Return Adjustment

To align our analysis with real-world scenarios, this study considers transaction costs incurred during portfolio management. According to Zerodha, a reputable stock broker, the transaction costs amount to 0.03% of the transaction value or Rs 40, whichever is lower. This cost structure is integrated into our calculations, ensuring that our findings reflect the financial impact of trading activities on portfolio returns. It provides a more accurate depiction of how transaction costs affect overall portfolio performance.

All analyses and computations described in this section were conducted using Python, a widely-used programming language for data analysis and visualization, and Google Colab, a cloud-based platform that provides a Jupyter Notebook environment for collaborative research. Python libraries such as NumPy, Pandas, Matplotlib, and Scikit-Learn were utilized to perform the various analytical tasks.

DATA ANALYSIS AND FINDINGS

Descriptive Analysis of Portfolio Constituents

This study delves into a comprehensive Descriptive Analysis of the portfolio constituents. This critical examination provides a detailed overview of the performance, trends, and key statistics of these prominent stocks. By analyzing their historical data and characteristics, we aim to gain valuable insights into the behaviour of these stocks, shedding light on their past and current dynamics, all while keeping the discussion concise and informative.

Statistical summary of portfolio constituents

Table 1: Statistical summary of individual stocks

Statistic	0P0001BA05.BO	BTC-INR	HDFCBANK.NS	HINDUNILVR.NS
count	2708	2662	2710	2710
mean	16.326	778730.54	694.554	1055.765
std	3.384	1165525.762	408.299	678.075
min	8.543	11058.415	181.977	210.961
25%	14.415	39761.382	311.335	484.571
50%	15.619	420739.593	559.544	774.759
75%	18.009	727221.593	1018.494	1625.938
max	24.269	49994456	1650.364	2686.178
Statistic	LT.NS	RELIANCE.NS	SETFGOLD.NS	TCS.NS
count	2710	2710	2708	2710
mean	929.391	858.47	2987.445	1327.782
std	330.545	614.102	716.209	798.342
min	364.048	307.102	1955.949	360.495
25%	617.315	408.822	2350.966	806.051
50%	896.075	490.499	2638.275	1076.883
75%	1194.629	1202.985	3032.212	1822.782
max	1930.814	2714.062	5027.25	3768.266

The descriptive statistics for the eight chosen assets, encompassing five Nifty 50 stocks, Bitcoin-INR, a gold ETF, and a thematic fund, offer valuable insights into their individual characteristics and potential portfolio contributions. Analyzing the data through the lens of established metrics like mean, standard deviation, and percentiles paints a picture of both central tendencies and potential for volatility.

I. Stock Price Comparison: TCS.NS emerges as the frontrunner in terms of absolute share price, boasting the highest mean and median among Nifty 50 constituents. Conversely, HDFCBANK.NS exhibits the lowest standard deviation, suggesting a relatively stable price compared to its peers.

II. Alternative Asset Volatility: As expected, the alternative assets are represented by Bitcoin-INR and SETFGOLD.NS, showcases significantly higher standard deviations compared to the Nifty 50 stocks. This underscores their inherent volatility but also potentially their ability to amplify portfolio returns during favourable market conditions.

III. Thematic Fund Performance: Thematic funds like 0P0001BA05.BO (DSP Natural Resource & New Energy Fund) holds promise with its relatively higher mean and median returns than the Nifty 50. However, their elevated standard deviation warrants further investigation into the risk-reward trade-off involved in their inclusion.

Calculation of daily mean returns

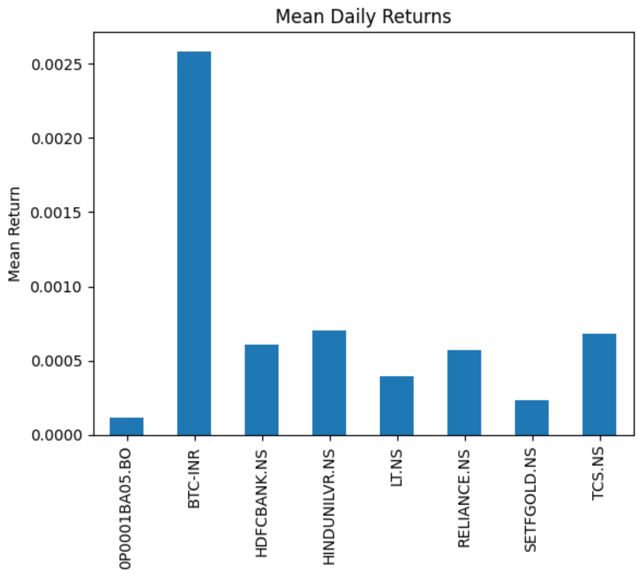

Fig 2: Mean Daily Return

I. BTC-INR stands out with its remarkable average daily return, suggesting significant potential for growth. However, this higher return comes at the expense of substantial volatility, evident in its highest standard deviation. This inherent risk should be carefully considered when analyzing its suitability for portfolio inclusion.

II. 0P0001BA05.BO exhibits a respectable mean return while maintaining moderate volatility, making it a potentially attractive option for seeking good returns with balanced risk exposure.

III. TCS.NS shines with its impressive Sharpe Ratio, indicating its ability to provide strong returns compared to its level of risk. This makes it a commendable candidate for a well-diversified portfolio aiming for efficient risk-adjusted performance.

IV. The remaining assets present a spectrum of varying mean returns and risk profiles, offering diverse options to tailor the portfolio according to individual risk tolerance and return objectives.

Portfolio Construction and Optimization

This subsection focuses on the construction and optimization of portfolios:

Monte Carlo simulation and Efficient Frontier plotting for portfolio optimization

In order to comprehensively evaluate the risk-return trade-offs inherent in our portfolio, we employed a Monte Carlo simulation, a powerful statistical technique that involves the generation of a large number of random portfolios. The objective was to explore a diverse range of portfolio weightings for the selected portfolio constituents.

A total of 50,000 portfolios were simulated, each with randomized weights assigned to the individual Nifty 50 stocks. This process allowed us to explore a vast array of potential portfolio compositions, considering different allocations to each stock. The simulation resulted in a diverse set of portfolios, each characterized by a unique combination of expected return, volatility, and Sharpe ratio. By plotting these portfolios on an efficient frontier graph, we identified the optimal portfolio—maximizing the Sharpe ratio, which represents the ideal balance between risk and return. The portfolio with the maximum Sharpe ratio was then extracted for further analysis.

Mathematical Optimization:

To complement the Monte Carlo simulation, a mathematical optimization approach was employed to identify the weights for the optimal portfolio with a maximum Sharpe ratio. The process involved formulating an objective function and constraints to be minimized, ensuring a practical and applicable solution. The mathematical optimization process yielded the precise weights for each stock in the optimal portfolio. These weights represent the allocation that maximizes the Sharpe ratio, providing investors with a strategic asset distribution to achieve an optimal balance between risk and return.

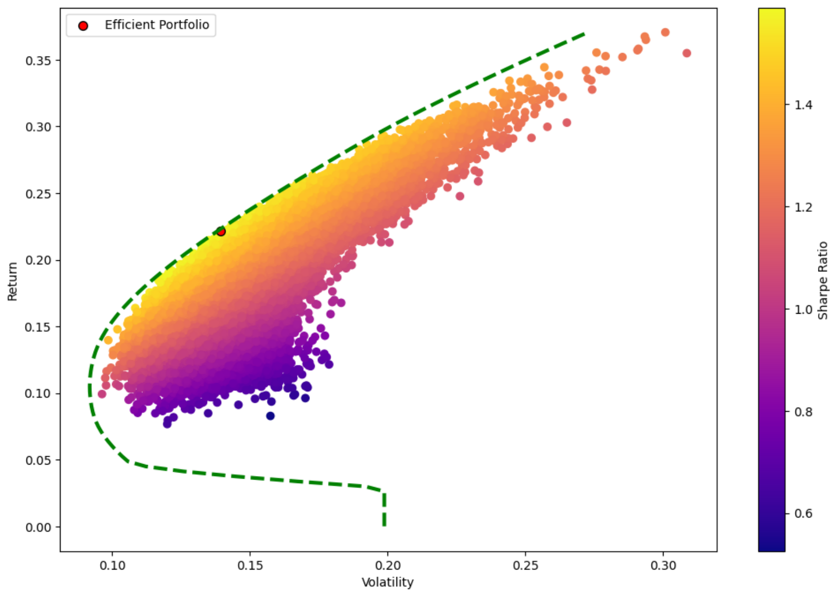

Fig 3: Efficient Frontier

Scatter Plot:

- Axes:
    - X-axis: Volatility, represented by the standard deviation of portfolio returns.
    - Y-axis: Expected return of the portfolio, annualized for 252 trading days.

- Points: Each point represents a simulated portfolio from the Monte Carlo simulation (50,000 in total).
- Colours: The colour of each point corresponds to its Sharpe Ratio, a measure of risk-adjusted performance (higher Sharpe Ratio indicates better performance relative to risk).

This analysis utilizes a 50,000-iteration Monte Carlo simulation to optimize portfolio composition within the Nifty 50 universe. The visualization depicts the resulting efficient frontier, highlighting the trade-off between expected return and portfolio volatility. The scatter plot displays a positive correlation between expected return and volatility - portfolios with higher returns tend to have higher risk (volatility). The efficient frontier forms a downward-sloping curve, highlighting the trade-off between risk and return. Portfolios closer to the frontier offer better risk-adjusted performance. The red dot, representing the optimal portfolio, rests on the efficient frontier, indicating it achieves the highest possible return for its level of risk. The distribution of points around the efficient frontier suggests that diversification can effectively reduce portfolio volatility without significantly sacrificing returns.

Table 2: Stock Weights of Efficient Portfolio

Sl. No	Stocks	Weights
1	0P0001BA05.BO	0.0020
2	BTC-INR	0.1691
3	HDFCBANK.NS	0.1207
4	HINDUNILVR.NS	0.2291
5	LT.NS	0.0107
6	RELIANCE.NS	0.0813
7	SETFGOLD.NS	0.2375
8	TCS.NS	0.1497

Black-Litterman Model Integration in Portfolio Optimization

This section explores the integration of the Black-Litterman (BL) model within portfolio optimization strategies. Our implementation leverages the PyPortfolioOpt library to construct efficient frontiers and identify optimal portfolio weights under both traditional Monte Carlo (MC) and BL frameworks.

Black-Litterman Model Implementation:

A custom function implements the BL model, incorporating investor risk aversion, uncertainty parameters, and views expressed through a prior mean (P) and prior covariance (Q). Historical data provides the basis for expected market returns, adjusted for daily frequency.

Efficient Frontier Optimization:

PyPortfolioOpt's EfficientFrontier object facilitates the calculation of both MC and BL frontiers. Within the BL framework, expected return estimates are adjusted based on the model's output. Subsequently, the maximum Sharpe Ratio portfolio is identified for each optimization approach.

Efficient Frontier Comparison:

A Monte Carlo simulation generates 5000 random portfolios for both MC and BL frameworks. Expected returns, volatilities, and Sharpe Ratios are computed for each portfolio, enabling the construction of their respective efficient frontiers. A visual comparison highlights the risk-return trade-off profiles under each methodology.

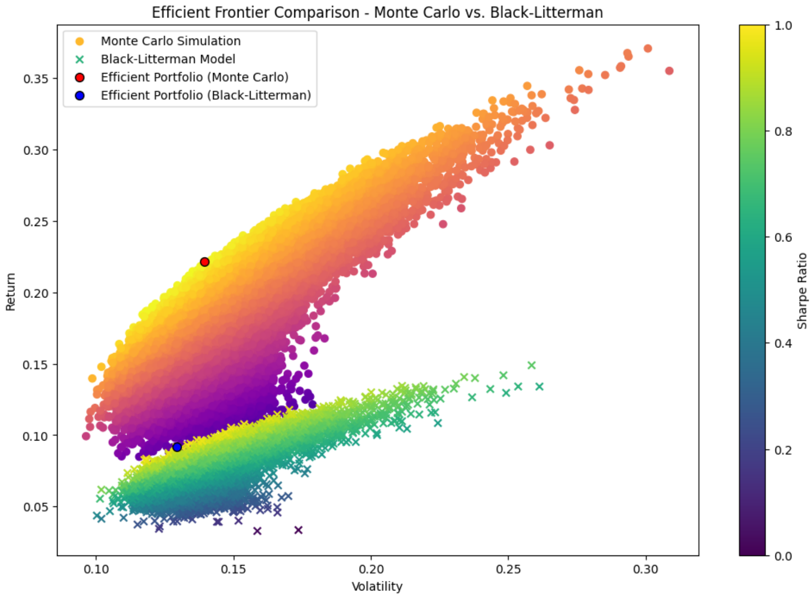

Fig 4: Comparative Analysis of BL Model and MC Simulation

Both frontiers exhibit a downward-sloping curve, reflecting the inherent trade-off between expected return and volatility. The MC frontier appears slightly steeper, suggesting potentially higher returns for increased risk compared to the BL frontier.

- Efficient Portfolio Locations:

  - MC Portfolio: The MC portfolio with the highest Sharpe Ratio (marked with a red dot) lies towards the upper right corner of the frontier. It offers an expected return of approximately 22.5% with a volatility of around 15%.

  - BL Portfolio: The BL portfolio with the highest Sharpe Ratio (marked with a green X) is located closer to the center-left of the frontier. It has a lower expected return of roughly 12.5% but also a significantly lower volatility of about 13%.

The differences in their risk-return profiles suggest that the BL model likely allocated assets in a more conservative manner compared to the MC approach, prioritizing lower volatility while sacrificing some potential return. This comparison highlights the potential impact of incorporating investor views through the BL model on portfolio construction. While the MC portfolio offers higher potential returns, it comes at the cost of significantly higher risk. The BL portfolio, on the other hand, prioritizes stability and risk mitigation, resulting in a lower expected return but also a considerably less volatile profile.

Portfolio Performance Evaluation

Construction of a dummy portfolio with an initial investment of Rs 100,000/-

To compare the performance of the efficient portfolio with the benchmark, an initial investment of Rs. 100,000/- has been taken to invest in the efficient portfolio according to the weights given by the algorithm.

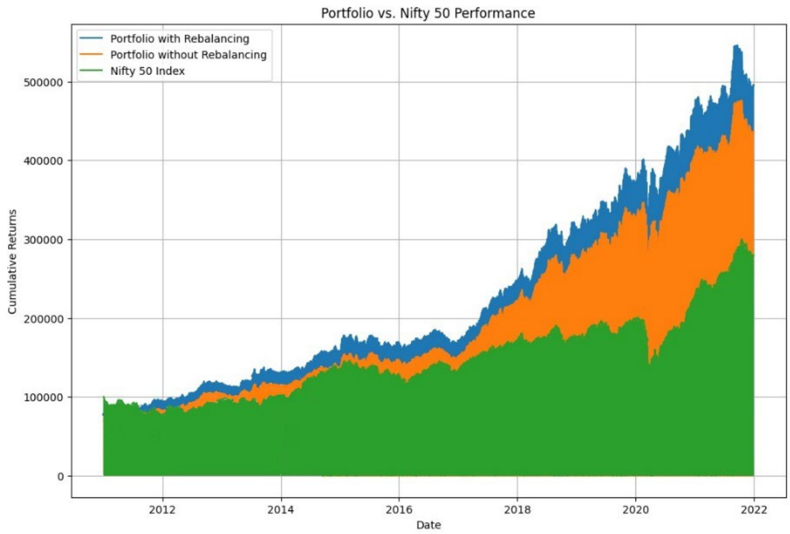

Fig 5: Comparative Performance of Portfolio with Rebalancing, Portfolio without Rebalancing, and Nifty50

- Superior Returns: Both portfolios significantly outperform the Nifty 50 index throughout the observed period, demonstrating the effectiveness of the initial optimal weight selection.
- Rebalancing Advantage: Notably, the rebalanced portfolio achieves higher cumulative returns compared to the buy-and-hold approach with static weights. This suggests that capturing market changes through periodic optimization can translate into enhanced performance.
- Volatility Trade-off: However, the rebalanced portfolio exhibits greater volatility, as evidenced by larger fluctuations in its cumulative returns. This highlights the inherent risk associated with frequent adjustments, which may not be suitable for all investors.
- Market-Dependent Performance: The benefit of rebalancing appears to be most pronounced during periods of significant market volatility ([years with wider performance gap]). During these times, adjusting asset allocations based on updated information can effectively capitalize on opportunities and mitigate potential losses. Conversely, in relatively stable markets ([years with narrower gap]), the performance gap diminishes, indicating that frequent rebalancing may not offer substantial advantages.

Table 3: Performance of Efficient Portfolios vs Nifty50

Metrics	Initial Value	Final Value	Annual Return	Absolute Return
Nifty 50 (index)	1,00,000	2,79,393.40	27.94%	279.39%
Portfolio without Rebalancing	1,00,000	4,36,477.83	43.65%	436.48%
Portfolio with Rebalancing	1,00,000	4,95,911.14	49.59%	495.91%

Both portfolios significantly outperform the Nifty 50 index in terms of absolute and annual return. The rebalancing strategy (49.59%) demonstrates the highest annual return, exceeding the buy-and-hold approach (43.65%) by nearly 6 percentage points. This gap suggests the effectiveness of dynamically adjusting asset weights based on updated market information.

Analysis of transaction costs and their impact

Incorporating a real-world perspective, this section delves into the impact of transaction costs on portfolio performance. The parameters for transaction costs were set at a transaction_cost_percentage of 0.03% (expressed as a decimal) or a minimum transaction cost of Rs 40 per order plus other charges like stamp duty, GST, etc, following the standards of Zerodha, a reputable Indian brokerage firm. In Table 4, here we can see the breakdown of the transaction cost of the portfolio during the period of study. In this period, a total of 39 rebalancing events occurred, with a total transaction cost of Rs 12,480. Within this context, we assess the tangible effects of transaction costs on portfolio returns. The outcomes reveal that transaction costs have a notable influence, reducing the average annual return of the Rebalancing portfolio compared to the non-rebalanced counterpart. This analysis underscores the significance of accounting for transaction costs in portfolio management, offering a practical understanding of their implications for investors in real-world scenarios.

Table 4: Breakdown of Transaction Cost

Year	Date	Transaction Cost	Year	Date	Transaction Cost	Year	Date	Transaction Cost
2011	31-Mar	235.94	2014	30-Sep	115.53	2018	30-Jun	373.18
2011	30-Jun	598.89	2014	31-Dec	191.65	2018	30-Sep	382.71
2011	30-Sep	461.11	2015	31-Mar	330.56	2019	31-Mar	107.42
2011	31-Dec	377.12	2015	30-Jun	272.1	2019	30-Jun	597.74
2012	31-Mar	98.28	2015	30-Sep	183.46	2019	30-Sep	608.29
2012	30-Jun	98.27	2015	31-Dec	385.43	2019	31-Dec	509.24
2012	30-Sep	545.64	2016	31-Mar	87.87	2020	31-Mar	191.89
2012	31-Dec	378.66	2016	30-Jun	184.03	2020	30-Jun	61.53
2013	31-Mar	446.04	2016	30-Sep	230.78	2020	30-Sep	461.02
2013	30-Sep	610.98	2016	31-Dec	287.29	2020	31-Dec	277.27
2013	31-Dec	524.39	2017	31-Mar	494.61	2021	31-Mar	76.88
2014	31-Mar	133.76	2017	30-Jun	125.78	2021	30-Sep	311.98
2014	30-Jun	114.54	2017	31-Dec	323.93	2021	31-Dec	572.81

Table 5: Net Return after transaction cost

Metrics	Transaction Cost	Net Value	Annual Return	Absolute Return
Nifty 50 (index)	133.77	279260.03	27.93	279.26
Portfolio without Rebalancing	238.04	436239.41	43.62	436.24
Portfolio with Rebalancing	12480.00	483431.14	48.34	483.43

The findings in Table 5 reveal the impact of transaction costs on the 3 portfolios:

- Transaction costs significantly impact the net returns of all three entities. Despite incurring the highest transaction cost (₹12,480), the rebalanced portfolio still achieves the highest net value (₹483,431.14) and annual return (48.34%). This highlights the potential performance benefits of rebalancing, even with transaction friction.
- The Nifty 50 index, with the lowest transaction cost (₹133.77), has the lowest net value and annual return compared to the portfolios. This demonstrates the potential outperformance of actively managed strategies, considering transaction costs.

CONCLUSION

In conclusion, our study delved into the risk-adjusted performance of a diverse portfolio in the Indian financial market, spanning a decade from January 1, 2011, to December 31, 2021. Our analysis included a mix of Nifty 50 stocks and new-age assets like Cryptocurrency, Gold ETF, and an energy-focused Mutual fund. Key findings like TCS.NS leading in absolute share price, with HDFCBANK.NS demonstrating stability. Alternative assets like Bitcoin and Gold ETF exhibited higher volatility but showed potential for amplified returns. The Monte Carlo simulations and mathematical optimization techniques were employed to construct portfolios, leading to the identification of an optimal portfolio on the efficient frontier. Integration of the Black-Litterman model provided a comparative analysis, highlighting the impact of investor views on portfolio construction. The optimized portfolios (Portfolio without Rebalancing: 43.65% annual return, 436.48% absolute return, Portfolio with Rebalancing: 49.59% annual return, 495.91% absolute return), despite substantial transaction costs ₹12,480., outperformed the Nifty 50 index, with the rebalanced portfolio showing higher cumulative returns. In summary, our research provides actionable insights for investors navigating the intricacies of the Indian financial markets. It underscores the importance of diversification, optimization techniques, and a nuanced understanding of risk-return dynamics in constructing resilient investment portfolios.

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