Brief Technical Note: A Markov Chain Approach to Measure Investment Rating Migrations

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Abstract

We explore two approaches (cohort versus hazard) to measure the probability of investment rate migrations of pension funds in Australia. We also develop validation procedures pertinent to each approach and find that the cohort method is more stable in its forecasts and reports a lesser migration probability to lower investment grades with minimal statistical significance. Conversely, the hazard approach reports a higher migration probability to lower investment grades with statistical significance. This finding has considerable consequences for fund managers as they seek to mitigate any downward trends in their investment appraisals, especially as the cohort approach is the industry's preferred approach in calculating rating migrations. The fund manager has a choice to make regarding measuring probability investment rate migrations, one between: stability (cohort) or accuracy (hazard).

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Introduction

The Australian population is expected to double from its current 22 million to 42.5 by 2056 (ABS, 2013) triggering fund managers to think more strategically about the financial sustainability of their pension funds. The Federal Government has engaged in an array of legislative provisions to monitor and safe guard such investments, with regulation compelling Australians to save for their retirement through workforce participation and compulsory contributions by employers (Iskra, 2012). However, it is clear that notwithstanding all the goodwill shown by the relevant authorities, no pension fund is immune to financial systematic risk. It is often of interest to accurately infer the expected investment performance rating of such funds for at least a one-year time horizon. The ability to structure future pension investment strategies based on expected investment performance rating probabilities is a valuable tool for investors/fund managers.

The literature advocates the use of the cohort and hazard rate approaches to determine the investment performance probabilities with transition matrices acting as outputs in estimating the migration probability to a lower/higher performance rating (Schuermann and Jafry, 2003). This inference is based on the observed historical ratings and as the cohort method is extensively used, it does not make full use of the available data with estimates unaffected by the timing and sequencing of the transitions over a pre-determined time. Calendar-year periods with overlapping 12-month intervals are the basis for such calculations, as this system assumes ratings remain stable, however reality shows us otherwise where market volatility is the norm. Consequently, this method downgrades high-grade investments over a relatively short period. This is a major concern, even more so as the industry employs this technique as their preferred calculation method. Rating migration probabilities are cardinal inputs to many investment decision applications, so accurate estimation is therefore required. A methodology that circumvents this problem is the hazard approach where within-period transition changes are captured providing more sequencing within the rating transitions (Jarrow and Lando, 1997).

To our knowledge, little empirical work on the application of transition probabilities on Australian datasets is recorded. In this paper we make a novel use of existing techniques on a dataset that is of significant importance to Australian pension fund holders, currently valued at $1.335 trillion (Industry Super Network, 2012). This is an increase from $1,170 trillion recorded in 2008 (APRA, 2009). The dataset was previously unavailable due to the lack of data providers, we now capture the entire retail pension funds in Australia and estimate their investment performance migration probabilities over a pre-determined period. We propose to initially employ the cohort method to measure the one- and two-year transition matrix migration probabilities. Furthermore, we construct one-year transition probability based on a generator matrix by applying an exponential function to the generator. Finally, we quantify our sampling errors by providing confidence intervals for the estimates and use the binomial distribution for the cohort approach and bootstrap confidence bounds for hazard to determine their statistical significance.

We find that indeed the empirical method to determine the transition probabilities of pension funds matters both statistically and from an investment performance decision as both methods yield different migration probabilities. The cohort approach provides more stable migration probabilities highlighting to investors no reason to switch to other funds, however such method is not statistically significant and investors could be using a less accurate model. Conversely, the hazard approach reports a high probability of investment downgrade with the results in the top three investment categories being statistically significant. So the choice of technique employed is between stability (cohort) or accuracy (hazard). This finding has implications both for fund managers and investors, as they seek to balance their risk/return expectations.

This study is structured as follows: Section 2 describes the estimation techniques for rating transitions, Section 3 reports the numerical testing and validation results with Section 4 concluding the study.

Estimation of rating transition or rating migration models

The initial methodology employed in this paper consists of estimating the transition probabilities of the investment performance rate changes of the pension funds by the cohort approach. Transition matrices are presented to demonstrate the transition probabilities of the funds from one investment performance rate to another over a period of time. The cohort approach uses the historical transition frequencies to estimate the frequencies in period t:

$$\hat{p}_{ij,t} = \frac{N_{ij,t}}{N_{i,t}} \tag{1}$$

where $N_{i,t}$ is the number of funds in rating i at the beginning of period t, i.e. the size of the cohort i, t; $N_{ij,t}$ is the number of funds from cohort i, t that have obtained rate j at the end of period t. When there are a number of periods being involved, there is a need to average the transition period frequencies using the weighted average technique as follows:

$$\hat{p}_{ij} = \frac{\sum_t N_{i,t}\hat{p}_{ij,t}}{\sum_t N_{i,t}} \tag{2}$$

Inserting (1) to (2) leads to

$$\hat{p}_{ij} = \frac{\sum_t N_{i,t}\frac{N_{ij,t}}{N_{i,t}}}{\sum_t N_{i,t}} = \frac{\sum_t N_{ij,t}}{\sum_t N_{i,t}} = \frac{N_{ij}}{N_i} \tag{3}$$

Hence in line with Hanson and Schuermann (2006) the fund-weighted average is equivalent to the division of the sum of transitions from rate i to j and the overall number of funds in rate i at the start of the considered period.

Furthermore, in line with Markov chain theory, we assume that the next state of transition is dependent on the current state of transition and is independent on the previous states. Given a sequence of random variables $X_1, X_2, X_3, \ldots, X_n, \forall n \in \boldsymbol{R}$, let $= \{s_1, s_2, \ldots, s_m\} = \{1,2, \ldots, m\} \ni X_n$, where S is the state space of Markov chain containing the possible values of X_n, $\forall n \in R$.

$$P(X_{n+1} = k | X_1 = j_1,\ X_2 = j_2,\ X_3 = j_3, \ldots, X_{n-1} = j_{n-1},\ X_n = j\ ) = P(X_{n+1} = k |\ X_n = j) = p_{jk}(n) \tag{4}$$

where $p_{jk}(n)$ is the probability of the transition from state k to j starting from time n to $n+1$, and this transition is called the single-step transition. In general, for all $j,k \in S$ and $n \geq 1$, the probability of transferring from state j to state k in n time steps starting from time 0 is defined as

$$p_{jk}^{(n)}(0) \equiv p_{jk}^{(n)} = P(X_n = k|X_0 = j), \qquad (5)$$

and the single-step transition is:

$$p_{jk}^{(1)} \equiv p_{jk} = P(X_1 = k|X_0 = j) \qquad (6)$$

The transition matrix is denoted by

$$P = (p_{jk}) = \begin{pmatrix} & \cdots & \\ \vdots & \ddots & \vdots \\ & \cdots & \end{pmatrix}$$

for a time-homogeneous Markov chain where the transition probabilities do not vary with time (Israel, Rosenthal and Wei, 2001). Conversely, a time-inhomogeneous Markov chain has transition probabilities that vary with time like for example mortality rates where time corresponds to age., for all $j,k \in S$ and, $n \geq 1$, the probability of transferring from state j to state k in n steps starting from time t is defined as

$$p_{jk}^{(n)}(t) \equiv p_{jk}^{(n)} = P(X_{t+n} = k|X_t = j) = P(X_n = k|X_0 = j), \qquad (7)$$

and the single-step transition is:

$$p_{jk} \equiv p_{jk} = P(X_{t+1} = k|X_t = j) = P(X_1 = k|X_0 = j) \qquad (8)$$

assuming that:

$P(AB|C) = \frac{P(ABC)}{P(C)} = \frac{P(A|BC)P(BC)}{P(C)} = P(A|BC)P(B|C)$, let's denote $l = 1,2,\dots,n-1 \in S$, and the n-step transition probabilities is established as follow

$$p_{jk}^{(n)} = P(X_{m+n} = k|X_m = j) = P(X_n = k|X_0 = j)$$

$$= \sum_i P(X_n = k, X_l = i|X_0 = j)$$

$$= \sum_i P(X_n = k|\, X_l = i, X_0 = j)\, P(X_l = i|X_0 = j)$$

by Markov chain assumption

$$= \sum_i P(X_n = k|X_l = i)\, P(X_l = i|X_0 = j)$$

and by the Chapmam-Kolmogorov equation

$$= \sum_i p_{ji}^{(l)} p_{ik}^{(n-l)} \qquad (9)$$

Hence before the transition matrices are obtained we first estimate a $n \times n$ generator matrix Λ giving a general description of the transition behaviour of the funds. The off-diagonal entries of Λ estimated over the time period $[t_{0,t}]$ are given as:

$$\lambda_{ij} = \frac{N_{ij}}{\int_{t_0}^{t} Y_i(s)ds} \text{ for } i \neq j \qquad (10)$$

where N_{ij} is the observed number of transitions from i to j during the time period considered in the study, and $Y_i(s)$ is the number of funds rated i at time s. Hence the denominator contains the number of years spent in rating class i. The on-diagonal entries are constructed as the negative value of the sum of the λ_{ij} per row

$$\lambda_{ij} = -\sum_{i \neq j} \lambda_{ij} \qquad (11)$$

and estimate transition probabilities following the Markov Chain Theory. The generator matrix Λ is then used to derive the T-year transition matrix $P(T)$ as follows:

$$P(T) = \exp(\Lambda T) = \sum_{k=0}^{\infty} \frac{\Lambda^k T^k}{k!} \qquad (12)$$

where ΛT is the product of the generator matrix and scalar T and *exp()* is the matrix exponential function.

As the transition probabilities in both methodologies are estimates they are subject to sampling error, therefore in line with Lando and Skodeberg (2002) we estimate the confidence interval as required to quantify the degree of sampling error. We use the Loffler and Posch (2011) binomial distribution approach to obtain the confidence intervals within the cohort method and bootstrapped confidence bounds for the hazard approach.

Numerical testing and validation results

The transition matrix algorithm is implemented in Visual Basic code and the numerical analysis is performed on the Australian pension funds dataset.

The pension fund credit rating market in Australia is mainly dominated by Thomson Reuters. In line with Pozen (2010), we employ MorningStar, a Thomson Reuters database to download the performance ratings of 1,829 Australian retail pension funds over the period 2001 to 2011. The investment performance ratings covering such funds are a relatively new feature and Thomson Reuters decided to make such information available due to its economic significance where retail pension funds increased by 11.5 per cent from \$1.198 trillion at end-June 2010 to \$1.335 trillion at end-June 2011. The performance investment ratings are a function of the funds' qualitative and quantitative characteristics, with 5 being the highest and 1 the lowest. This feature is in contrast to the U.S. where seven broad rating categories are recorded. Our focus is on the one-year time horizon as that is typical for many credit applications. However, the longer the horizon the more migration potential. The database has a total of 14,688 obligor years of data excluding withdrawn ratings of which 48 ended in default. On average, 73% of the dataset constitutes investment grade 3, 4 and 5.

Cohort approach: As there are five possible investment performance rating grades in our study, the transition matrices are in 5 × 5 dimensions. Table 1 shows the one- and two-year investment performance transition probabilities for Australian pension funds.

Table 1:

One- and two-year transition probabilities using the cohort approach

Panel A					
1 yr trans.	5	4	3	2	1
5	61.50%	33.80%	4.69%	0.00%	0.00%
4	10.44%	65.26%	19.48%	4.82%	0.00%
3	0.69%	19.38%	63.84%	15.92%	0.17%
2	0.00%	1.81%	23.53%	73.53%	1.13%
1	0.00%	0.00%	0.00%	0.00%	100.00%
Panel B					
2 yr trans.	5	4	3	2	1
5	41.39%	43.76%	12.47%	2.38%	0.01%
4	13.37%	49.98%	26.77%	9.79%	0.09%
3	2.89%	25.54%	48.31%	22.80%	0.46%
2	0.35%	7.07%	32.67%	57.90%	2.00%
1	0.00%	0.00%	0.00%	0.00%	100.00%

5 is the highest investment performance grade and 1 is the lowest.

Table 1 Panel A employs the cohort approach and reports the on-diagonal entries being the highest on a one-year transition matrix in the range of 61.50% down to 100% on the worst rating. Overall, the matrix suggests a relatively stable rating system as the probability to other ratings is rather low. Moreover, the performance downgrades to the two lowest two rating categories, namely Category 1 and 2 are 0.00%. Nevertheless, we would still expect that the performance downgrade to be very small as such funds are highly regulated with transparent investment strategies. We extend further the transition probability of Australian pension funds over a longer period than one year. We assume that the transitions are independent across the years and Panel B in Table 1 reports the transition probabilities over two years. Overall, the on-diagonal entries remain the highest but are less compared to the one-year transition matrix and range between 41.39% to 100%. It is evident that as one extends the transition further in time, the accuracy levels become less reliable. Our focus remains the one-year horizon as that is typical for many credit applications.

Hazard approach: In view of the limitations applicable to the cohort method, the hazard approach is applied and Panel A in Table 2 represents the generator matrix employed in providing a general description of the transition behaviour. The off-diagonal entries are estimated by Equation 10, with the denominator containing the time spent in rating class *i*. This is similar to the cohort method, where we count the funds at discrete points in time. In the hazard approach we count the funds at any point in time. The on-diagonal entries are constructed as negative values of the sum of the fund per row. By applying the exponential function to the generator, we report a one-year transition matrix.

Table 2:

A one-year transition matrix derived from the generator

Panel A					
Generator	*5*	*4*	*3*	*2*	*1*
5	*-1.708*	*1.686*	*0.014*	*0.005*	*0.000*
4	*0.608*	*-1.972*	*1.359*	*0.004*	*0.000*
3	*0.004*	*1.057*	*-2.251*	*1.184*	*0.007*
2	*0.002*	*0.002*	*1.440*	*-1.585*	*0.140*
1	*0.000*	*0.000*	*0.077*	*0.309*	*-0.387*
Panel B					
1 yr trans.	*5*	*4*	*3*	*2*	*1*
5	*28.51%*	*39.14%*	*22.77%*	*8.94%*	*0.49%*
4	*14.11%*	*36.44%*	*31.69%*	*16.56%*	*1.17%*
3	*6.38%*	*24.62%*	*37.37%*	*28.62%*	*3.00%*
2	*3.05%*	*15.66%*	*34.94%*	*39.47%*	*6.87%*
1	*0.53%*	*3.40%*	*10.59%*	*16.29%*	*69.18%*

5 is the highest performance grade and 1 the lowest.

The on-diagonal entries in Panel B of Table 2 show the rating categories are still the highest, besides the investment performance grade 5 funds. However, on average they are less than that of cohort approach. Furthermore, we observe that there is nonzero performance downgrade for each fund rating categories. This indicates that even when the fund is ranked as the highest investment grade, it is also at risk of being downgraded to the lowest rating grades. For example, there is a probability of 39.14% for 5-star rated funds to be downgraded to 4-star rated funds in a year; and there is a probability of 31.69% for these funds to be downgraded to 3-star rated funds in another year. In a year's time, there is a 28.62% probability of these funds downgraded to 2-star rated funds. The process continues until the funds underperform and hence leave the study. Therefore, we will also record a probability of default for the highest investment grade funds.

Clearly the one-year transition matrix extracted from the cohort method differs to the hazard approach with the former providing higher probabilities that top rated investment funds are more likely to retain their current rating. The hazard migration probabilities are more pessimistic and record higher investment rate downgrades for the top rated investment funds. Clearly this inconsistency is a concern and in line with the literature (Shermann and Jafry, 2003) as both methods are based on Markov and time-homogeneity assumptions they still contrast mainly in that they are expressed in a discrete- and continuous-time framework, respectively. There is no specific explanation to this phenomenon but the sequencing of the rating transitions has been completely ignored in the cohort approach. Further testing is required and validation techniques provide robustness to the analysis.

Validation techniques: The transition matrices in both methods are estimates of transition probabilities and like all estimates they are affected by sampling errors. Therefore to ensure that the outputs are statistically significant within a 0.05% level we calculate confidence intervals. We implement a binomial distribution for obtaining confidence bounds for the cohort approach. As we assume that the rating downgrades are independent across time and across funds, we allocate the downgrades in a binomial distribution with N_i successes and success probability PD_i. and subsequently derive confidence bounds. In reality rate downgrades are not independent, however it is a good starting point to extract confidence bounds. Therefore we seek a two-sided, 1-α confidence interval where α is set to 5% with a lower bound PD_i^{min} where the probability of observing N_i rating downgrades or more is α/2 and is represented as follows:

$$1\text{-BINOM}\ (N_{ik} - 1, N_i, PD_i^{min}) = \alpha/2 \quad (13)$$

where BINOM (x,N,q) denoting the cumulative binomial distribution for observing x or less success out of N trials upgrades probability q. Furthermore, the upper bound PD_i^{min} has a probability of observing N_i or less downgrades is α/2 and is represented as follows:

$$\text{BINOM}\ (N_{ik}, N_i, PD_i^{max}) = \alpha/2 \quad (14)$$

Therefore we construct the confidence intervals in Table 3 with the PD_i^{min} and PD_i^{max} describing the confidence bounds for the cohort method. Columns designated as *Equation* * and ** set the conditions for the confidence sets in accordance with Equations 13 and 14.

Table 3:

Binomial confidence bounds for investment grades from the cohort approach

Performance ratings	PD_i^{min}	PD_i^{max}	*Equation**	*Equation ***
5	0.00%	2.18%		
4	0.00%	0.37%		
3	0.00%	0.21%	0.000%	0.000%
2	0.15%	0.96%	0.000%	0.000%
1	1.67%	3.51%	0.000%	0.000%

*Equation**: $1 - \text{BINOM}\ (N_{ik} - 1, N_i, PD_i^{min}) = \alpha/2$; *Equation***: $\text{BINOM}\ (N_{ik},\ N_i, PD_i^{max}) = \alpha/2$ with α is the significant level set as 5%.
This test is two-sided, $1 - \alpha$ confidence interval where $\alpha = 0.05$; PD^{min} is the lower bound and must be such that the probability of observing N_i or more defaults is $\alpha/2$, therefore solves the condition for *Equation**; PD^{max} is the upper bound and must be such that the probability of observing N_i or less defaults is $\alpha/2$, therefore solves the condition for *Equation***.

The confidence intervals recorded under columns PD_i^{min} and PD_i^{max} are relatively wide and in most cases there is a high commonality with their respective performance rating grades. This result suggests that the migration probabilities are not statistically significant and the top performing funds have a wider confidence interval (0.00 to 2.18) than the lower grades. The widest intervals are reported in the lowest rating category and this result is expected as there are a relatively small number of funds in this category compared to the other categories.

As it is not clear how to apply the binomial distribution method to the hazard approach, we bootstrap a number of simulations and derive a distribution of the statistic of interest. We randomly draw with replacement a fund's complete rating history and repeat for 1,829 times representing the amount of funds within the dataset. We then calculate the generator ∧ and transition matrix *exp*(∧) for the sample generated. This process is repeated 1,000 times over until we finally determine the percentiles of the transition probabilities and calculate the confidence for the probability of rating downgrade with 5% confidence as reported in Table 4.

Table 4:

Bootstrapped confidence bounds for performance downgrade probabilities from the hazard approach

Performance ratings	Lower	Upper
5	0.00%	0.01%
4	0.00%	0.01%
3	1.02%	0.23%
2	3.71%	7.92%
1	10.62%	24.91%

The validation tests for the hazard approach present results different from their counterparts within the cohort method, where performance ratings 5 and 4 report a narrow confidence limit suggesting that the transition probabilities are statistically significant.. A range of 0.00% and 0.01% with 95% confidence is reported. Conversely the intervals widen as the performance rating decline further. Same as the cohort method, the lower investment performance ratings are bound to be inaccurate due to their low number of funds in that investment category.

Concluding remarks

In this paper we presented two estimation methods for investment grade migration matrices for Australian pension funds - the cohort and hazard approach. We ask which method provides more accurate estimations by introducing validation techniques for each method. As we investigate the mobility matrix (migration matrix $\boldsymbol{P}$ less the identity matrix $\boldsymbol{I}$ of the same size) we find that indeed the rating method matters both statistically and from an investment performance decision.

The two methods yield statistically different migration matrices. The cohort approach provides more stable migration probabilities highlighting to investors no reason to switch to other funds, however such method is not statistically significant and investors could be using the less accurate model. Conversely, the hazard approach reports a high probability of investment downgrade with the results in the top three investment categories being statistically significant. So the choice of measuring probability rate migrations is one between stability (cohort) or accuracy (hazard). This finding has implications both for fund managers and investors, as they seek to balance their risk/return expectations.

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