



A New Method to Estimate NPV and IRR from the Capital Amortization Schedule and the Advantages of the New Method

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Abstract

This paper introduces a new method, different from the discounted cash flow (DCF) method, for the first time, to estimate NPV and IRR. This method makes use of the capital amortization schedule (CAS). Accordingly, the present value (PV) of the closing balance (CB) in a CAS at a particular discount rate is the NPV at that rate and the interest rate that makes the CB zero is the IRR. CAS method also reveals that NPV represents the unutilised net cash flow (NCF) that remains as the CB in CAS. IRR is the only rate that fully utilize the NCF and makes the CB zero. The estimated NPV and IRR by CAS and DCF methods are perfectly matching in all cases of small or large-scale investments and under the financial and or economic analysis of investments. The CAS method is more transparent than the DCF method and provides a better insight into: a. evidence of reinvestment of intermediate income in some normal NCF and most non-normal NCF (NNCF) investments; b. elimination of the reinvestment income to get a unique IRR that resolves the problem of multiple IRR; and c. to identify the appropriate criterion between IRR and NPV, as NPV indicates the unutilised NCF whereas IRR indicates the return on invested capital (ROIC) by fully utilizing the NCF. The investors are comfortable to compare the IRR with the cost of capital in percentage term. As the modified IRR (MIRR) assumes reinvestment, MIRR might become redundant if there is no reinvestment and this is an incidental inference drawn.^{2,3}

JEL: C60, C63, D61, E22, E40, G3, G24, G31, H43, O2, O12, O16, O2

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1. Introduction:

The most popular DCF method for CBA and capital budgeting has been considered as “ambiguous or anomalous” as early as 1956 (Lorie and Savage). Recently Merlo (2017) argued that the DCF methodology cannot be a correct approach in some cases. Most published works attributed the multiple IRR as one of the limitations of IRR but not as a problem with the DCF method. Reinvestment and multiple IRR are, inter alia, the symptoms of the problems with DCF method and the underlying causes are not hitherto completely evaluated. As these symptoms often reflect on the IRR, the IRR is considered as inferior to NPV. There is no research evidence available to support that assertion.

Despite some of the shortcomings with the current discounted cash flow (DCF) method, there is no other better method to estimate NPV and IRR. Published papers in this area do not indicate any other suitable method of worth mentioning. Generally, economists and finance professional have been using the DCF method with NCF data to estimate the IRR and NPV.

The method presented in this paper is based on capital amortization schedule (CAS) prepared using the NCF data. CAS concept is based on the loan amortization schedule. Amortization is a process of recovery of a loan or investment capital from the future income or payments. CAS is a complete table of periodic payments or income from an investment to liquidate the capital invested (return of capital – ROC) at a required interest or return⁴ (return on capital invested - ROIC). CAS can be easily prepared from the NCF estimate. Banks and financial institutions have been using extensively the loan amortization schedule and the CAS is simply an extension of the cash flow analysis.

The purpose of this paper includes:

- a. introducing a new method based on the CAS to estimate NPV and IRR;
- b. comparing the estimated results with that of DCF estimates;
- c. illustrating how the new method provides a better insight to understand the problems associated with the controversial reinvestment assumption, multiple IRR;
- d. to identify the appropriate criteria (NPV or IRR) for capital budgeting, cost-benefit analysis and capital investment analysis (CIA); and
- e. estimating IRR on total capital

2. Review of Literature:

Capital investment decision involves estimation of two components of return viz. ROC or recovery of capital and a desired ROIC. The ROC and ROIC measures the return earned on capital invested (Damodaran, 2008). Return of capital (ROC) refers to principal (invested capital) payments back to "capital owners" (shareholders, partners). The return on invested capital (ROIC) is the percentage return that investors make over their invested capital. The investor must get back the

⁴ Discount rate or interest rate accounts for the rate of return required by the investor (to cover costs, risks and lost opportunities) and to account for the time value of money (TVM).

principal amount or the capital invested (ROC) and a return on invested capital (ROIC) as earning on investment.

Crean (2005) recommended to calculate an investment's "amortization" or "capital recovery" schedules that are a function of the magnitude and timing of the investment's cash flows. In this paper CAS, in the form of capital amortization schedule, is used. CAS has been extensively used by Banks or borrowers to arrive at the loan repayment schedule for a given interest rate regime.

Nobel prizewinners Franco Modigliani and Merton Miller identified return on investments as a major component of value creation. Unless company's return on capital exceeds its cost of capital, no amount of revenue growth can create value (Marco-Izquierdo, 2015). Jiang and Koller (2007) reported that although growth is good, returns on invested capital (ROIC) can be an equally—or still more—important indicator of value creation.

Berkovitch and Israel (1998) showed plausible situations where the NPV criterion leads to inefficient capital budgeting outcomes. They also showed that the IRR and PI (PI = profitability index = Benefit Cost Ratio - BCR) are useful in curbing empire-building managers because, when selecting between mutually exclusive projects, they tend to bias against largescale projects.

Jacobs (2007) discussed that 'the NPV-method and the IRR-method are not two measures of investment worth - as it is reported in many textbooks - but just one single method. Moreover, the NPV/IRR-method is plain mathematics and does not pretend to be a ranking device; it cannot be if the tool is used properly, and the results are interpreted correctly'. Jacobs comments used as such either. Mathematics is, yes, indeed a tool, but economics can only then be the master are valid as NPV and IRR are estimated using the NCF data and the DCF method and not two methods.

The DCF method is often criticised because of the controversial assumption of reinvestment of intermediate income. Many authors argued that the DCF method involves reinvestment (e.g., Shirvani & Wilbratte, 2009; Arnold & Nixon, 2011; Walker et al. 2010, Kierulff, 2012) and others rejects the reinvestment assumption (e.g., Johnston et al., 2002; Ross et al. 2008; Rich & Rose, 2014; Schmidt, 2014). In recent research, Arjunan and Kannapiran (2017) simulated with and without reinvestment scenarios. The simulated NCFs are used in their investigation. They concluded that there is no reinvestment of intermediate income under the DCF method and in CBA. Arjunan (2017a), using a CAS method (like the one discussed here), illustrated that the CAS method transparently reveals that reinvestment of intermediate income does occur only with some of the non-normal NCF (NNCF) and that such reinvestment leads to multiple IRR. This is an important finding that is explained by the CAS method.

Some texts discuss the traditional pay-back period and the accounting rate of return technique as alternatives to the DCF method. These traditional approaches fail to account for the time value of money (TVM) and cannot be considered as alternate methods even though they are easy to estimate and popular. Over the years, several authors attempted to introduce a modified or alternative method. Almost all of them left their footprints in the form of new terminology to the DCF method or capital budgeting without substantial improvement. Some of those methods are briefly discussed.

Predominantly, MIRR is recommended as a preferred method that might overcome the common problems of reinvestment, multiple IRR and ranking of mutually exclusive investments (see Brigham and Ehrhardt, 2016; Ross, 2015; Balyeat et al, 2013; Keirulff, 2008). Arjunan

(2017b) analysed the MIRR method and concluded: a. MIRR is not consistent with the actual NCF (i.e. in some cases, the given NCF is not fully utilized by the MIRR and in other cases the NCF is not adequate to support that MIRR); b. illustrated how the MIRR method distorts the intrinsic value of the cash inflow and its timing; and c. concluded that MIRR is a spurious estimate that increases with increase in investment rate for the given NCF.

Weber (2017) introduced a selective IRR (SIRR) criterion that he claimed to be equivalent to the NPV-rule. He argued that “an investor with a cost of capital of $r = 10\%$ would report the return of a project with the cash-flow stream (-5, 16, -12) as minus infinity (and therefore completely unacceptable), whereas an investor with a cost of capital of $r = 25\%$ would report the return of the same cash-flow stream as 100% which is very attractive indeed.” The 100% return is highly unrealistic and purely a mathematically generated return and not by the project benefit stream. The cumulative undiscounted NCF is -1 (16 minus 17 leads to a cumulative loss of 1) and that being the case no investor will be interested in such a project. SIRR is therefore not a realistic estimate.

Magni (2010) introduced the Average IRR (AIRR, also Aggregate Return on Investment - AROI) and estimated an AIRR of -27.27% for a NCF data (Magni, *ibid*, page 5, Table 1: NCF: -10, 30, -25). An AIRR of -27.27% is not feasible when the cumulative NCF leads to a net loss of 5 (30-35 = -5) i.e., a capital loss of 50%. When the capital loss is 50%, the estimated IRR must be around -50% and not -27.27%. The AIRR of -27.27% is not consistent with a capital loss of 50% and therefore it is unreliable.

Kulakov and Kastro (2015) discussed two estimation methods for NNCFs viz. a generalized internal rate of return (GIRR) for a project as an investment; and a generalized external rate of return (GERR) for a project as a loan. They used the famous ‘oil pump installation project’ NCF data studied by Solomon (1956, see Table:1, NCF: -1600, 10000, -10000 of Kulakov and Kastro, *ibid* page 5) with a cumulative loss of -1600 (capital loss of 100%). That being the case, the rate of returns (GIRR or GERR) of 16.9% estimated looks unrealistic. These methods have not made any improvement and a detailed analysis is available in Arjunan (2017a).

Mackie et. al. (2005) discussed about an Adjusted IRR (AIRR) and indicated that the IRR will overstate the true rate of return, and the Adjusted IRR, with appropriate reinvestment rate, will give an estimate of the true value. This finding again depends on the doubtful reinvestment assumption (see Arjunan and Kannapiran, *ibid* page 4).

There is limited number of published works relating to useful alternate methods as there is apparently very little research done in this area. This paper contributes substantially to fill that gap and presents a totally different approach to estimate the NPV consistently and transparently and IRR based on CAS data.

3. Methodology

First, the DCF analysis, using NCF data, is discussed to highlight the relationship with the new method. The DCF method commonly uses Equation 1 to estimate the IRR and NPV.

$$NPV = \frac{CF_1}{(1+r)^1} + \frac{CF_2}{(1+r)^2} + \dots + \frac{CF_t}{(1+r)^t} - CF_0 \quad \text{Equation 1}$$

Where, $CF_1, CF_2 \dots CF_t$ are the NCFs during periods 1 to t (t is the final or terminal year); CF_0 is the capital invested; 'r' is the discount rate. NPV is estimated with hurdle rate or cost of capital as the discount factor 'r'. IRR is identified by using a range of discount rates (r). The 'r' that makes the $NPV = 0$, is the IRR.

In Eq.1, the sum of the right-hand side (RHS) variables (other than the CF_0) is the PV of NCFs (PVCF). PVCF minus CF_0 is the NPV as shown in Eq.2 that is derived by substituting PVCF on the RHS in Eq.1.

$$NPV = PVCF - CF_0 \tag{Equation 2}$$

This equation is important to understand or interpret the NPV. First, as per Eq.2, NPV is the return on investment in absolute terms (ROIC in \$). NPV also represents the balance of the PVCF after recovery of the capital (CF_0 or ROC) and cost of capital (at hurdle rate or cost of capital). The PVCF keeps changing for every change in the discount rate (r). The discount rate used represents the return on invested capital (ROIC) in percentage terms. ROIC is the return or interest rate earned on the balance of capital invested. As such, the discount rate (ROIC) used in the denominators of each present value (PV) computation is critical in determining what the final NPV number will turn out to be. A small increase or decrease in the expected or desired ROIC (r) will have a considerable effect on the final output of NPV. Several interpretations can be made from the Eq.2 as below:

When $r = IRR$, the NPV will be '0' that indicates the full utilization of the PVCF to pay-off the CF_0 (ROC) and the highest possible ROIC (= IRR). With IRR as the ROIC, the investment income is optimized.

When $r < IRR$, the NPV is positive. The positive NPV represents the unutilized PVCF (see Eq.2). A lower discount rate (r less than IRR) is inadequate to maximize the ROIC ($ROIC < IRR$).

When $r > IRR$, the NPV is negative. Here, the PVCF is not adequate to support a higher ROIC (r higher than IRR) and therefore the NPV is negative. PVCF, being an important variable, can also be derived easily from CAS and therefore CAS is prepared.

Capital amortization schedule (CAS): Capital amortization schedule is a table or chart showing how much of each periodic future income or return from an investment is going towards interest payments or ROIC and ROC. CAS analysis indicates when the NCF is fully utilized (closing balance (CB) zero) to recover ROC and the ROIC. CB of >0 or <0 reveals that there is unutilized (excess) benefit at that ROIC or benefits not sufficient to support the required ROIC, respectively.

Under the DCF method, NCF is discounted at a rate that accounts for the cost of capital, return on investment and time value of money. In a CAS, the interest rate or the required return, accounts for the recovery of the ROIC (to cover the cost of capital, return on investment and TVM). The NCF is apportioned towards recovery of the ROC and ROIC and the balance is the final or terminal year CB. CAS estimation follows equation 3.

$$\sum_{t=1}^n (OB_t(1+r) + CF_t) = CB_t \tag{Equation 3}$$

Where OB is the opening balance of capital each year (- CF₀ in year one), r is the interest rate (or return r or discount), CF_t is the net cash inflow at the end of each year, CB is the closing balance in each year and 't' is year 1 to n (n is terminal or final year of project life or maturity of investment). The CB is sum of the opening balance of capital (OB), the interest or return paid and the CF_n each year. The CB of capital in a year is the opening balance (OB) of capital in the next year. The final or terminal year CB is the balance of the NCF after ROC and ROIC in \$. As per Eq. 1, NPV in year 1 as in Eq. 4.

$$NPV = \frac{CF_1}{(1+r)^1} - CF_0 \quad \text{Equation 4}$$

Now, obtain the present value of the CB in the CAS by dividing the CB (eq.3) by (1+r)^t as in Eq. 5 where 't' is years 1 to t.

$$PVCB = (-CF_0(1+r) + CF_1) / (1+r)^1 \quad \text{Equation 5}$$

The simplified Eq. 5 reveals that the PV of the CB is the NPV (as defined in Eq.6) during year 1.

$$PVCB = \frac{CF_1}{(1+r)^1} - CF_0 = NPV \quad \text{Equation 6}$$

Thus, NPV can be estimated by estimating the PV of the final or terminal year CB, without the use of DCF method. The interest rate that makes the closing balance zero is the IRR (Eq. 7).

$$\sum_{t=1}^n (OB_t(1+r) + CF_t) = CB_t = 0 \quad \text{Equation 7}$$

IRR can also be estimated by interpolating two consecutive NPVs (estimated using Eq. 6) that have been estimated by using two random, but closest, interest or discount rates in an iterative process until one of the NPV is positive and the next closest NPV is negative. Then using those NPV values and discount rates, IRR is estimated by linear interpolation as discussed here.

$$\text{Internal Rate of Return} = R_1 + [(NPV_1 \times (R_2 - R_1)) / (NPV_1 - NPV_2)]$$

Where: R₁ = Lower discount rate; R₂ = Higher discount rate;

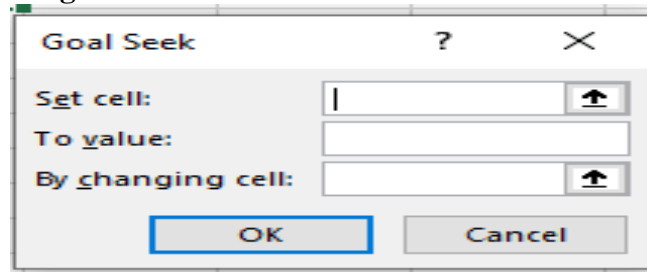
NPV₁ = Higher NPV (derived from R₁); NPV₂ = Lower NPV (derived from R₂);

Mathematically, IRR is the rate that makes the NPV zero. When NPV = 0, as per Eq. 6, the discount rate or interest rate makes the NCF = CF₀ (ROC) and the interest rate is equal to IRR (ROIC). Theoretically, NPV is the unutilized sum of income from an investment after recovery of the cost of capital at hurdle rate (say at 10%) and the ROC. NPV is the ROIC in \$ or

absolute term whereas the IRR fully utilize the income flow to express the ROIC in percentage term. The CAS truly reflects these relationships.

The estimation of IRR and NPV using the CAS can be easily carried out in Excel. IRR is the interest rate that makes the final CB zero. IRR can be estimated simply by using the “goal seek” function in the excel (see figure 1).

Figure: 1. Screenshot of the Goal Seek function in Excel



In Excel the “Goal seek” function can make the CB = 0 by changing the interest rate by an iterative process, for a given interest/discount rate, ONLY if the CAS is built with iterative formulae. In excel Go to Data > What if Analysis: In the dialog box (Figure 1) - "Set cell" hold the final CB cell; "To value" = 0 (to make CB = 0); and "By changing cell" = interest/discount rate % cell. If there are suspected multiple IRR (when the NPV function is non-monotonic⁵), the CB will be zero as many times as there are IRR (multiple IRR). Similarly, when the CAS is prepared, using the desired interest or discount rate or hurdle rate (r), then PV of the final CB = NPV (= CB_t / (1+r)^t).

BCR is the ratio of present value (PV) of benefit to the PV of cost. The CB is the excess of income after the recovery of CF₀. Therefore, CF₀ + PV of CB = PV of benefit; The PV of the CB is the NPV. CF₀ is the PV of capital cost. BCR is estimated using the formula, (CF₀+NPV)/CF₀ and the reduced form of this equation is Eq 8:

$$BCR = 1 + \left(\frac{NPV}{CF_0} \right) \quad \text{Equation 8}$$

The methodology also includes some investment project NCFs available in public domain and two real life projects (in Vietnam and Laos) funded by the Asian Development Bank (ADB). The DCF estimated IRR and NPV of those projects are compared with estimates by CAS methods. There are three purposes for such comparison: i. to illustrate whether the CAS estimated results are consistent both for small and large projects (with NCF for 25 to 40 years); ii. to demonstrate that the CAS method is appropriate both for NCF and NNCF projects; and iii. to reveal that the model estimates are universal for financial and economic IRR and NPV estimates.

In the next section, the new method to estimate the NPV and IRR is illustrated with numerical estimates and compared with the estimated NPV and IRR by DCF method in Excel.

⁵ A non-monotonic function increases and decreases with increasing discount rate.

4. Results and Discussions:

The focus of this analysis is to demonstrate the robust nature of the new estimation method using CAS data that provides a transparent and better insight into the NPV and IRR estimates. The presentation includes:

- a. numerical illustration of CAS method by making use of NCF data from different projects that are available in public domain and real-life projects including two non-normal NCF (NNCF) projects;
- b. comparison of the estimated results of the CAS method with that of the DCF method to illustrate that the estimates are appropriate for all types of projects;
- c. the CAS method is also tested with two real-life large projects with 25 to 40 years of life that were funded and appraised by the Asian Development Bank in Laos and Vietnam. The estimated results are compared with the DCF estimates. The comparison is also made with both financial and economic IRR and NPV to illustrate the results by CAS and DCF methods are consistent in all cases.
- d. demonstration of how the new method transparently reveals the reinvestment of intermediate income that leads to multiple IRR, how to eliminate multiple IRR and to identify the appropriate criterion (NPV or IRR) for CIA and CBA;

The results from all these analyses are presented in tables 1 to 5 and discussed in this section.

4.1 CAS based Estimation Method and Comparison with DCF Estimates:

The NCF data is the starting point for both DCF and CAS methods and there is no difference in estimation of NCF for investment projects. The negative components of a NCF are the investments and the positive components are the net income flow (after operating costs). The estimated IRR and NPV by using the CAS method and presented in table 1 and 2 and discussed.

- a. First CAS is prepared (Table 1) for selected investment projects at various discount rates or interest rates (ROIC) that include 10% (assumed hurdle rate) and a range of higher or lower rates that make the CBs zero or negative or positive. The new method introduced in this paper makes use of the numerical relationship between the CBs in CAS and the NPVs (see Eq. 6) and IRRs (see Eq.7) to estimate the NPV and IRR without resorting to the DCF method.

Table:1. CAS for two normal NCF and two non-normal NCF projects

Years	Opening Balance (OB)	Interest Charged (IC)	Income flow (CF)	Closing Balance (CB = OB+IC+CF)	Opening Balance (OB)	Interest Charged (IC)	Income flow (CF)	Closing Balance (CB= OB+IC+CF)
Normal NCF - Project A - at 10% Interest				Normal NCF - Project B - at 10% Interest				
0			1000				-1000	
1	-1000	-100.0	0.0	-1100.0	-1000	-100.0	320.0	-780.0
2	-1100	-110.0	0.0	-1210.0	-780	-78.0	320.0	-538.0
3	-1210	-121.0	300.0	-1031.0	-538	-53.8	320.0	-271.8
4	-1031	-103.1	600.0	-534.1	-272	-27.2	320.0	21.0
5	-534.1	-53.4	900.0	312.5	21	2.1	320.0	343.1
Normal NCF - Project A - at 15% Interest				Normal NCF - Project B - at 20% Interest				
0			-1000.0	194.0			-1000	213.1
1	-1000.0	-150.0	0.0	-1150.0	-1000.0	-200.0	320.0	-880.0
2	-1150.0	-172.5	0.0	-1322.5	-880.0	-176.0	320.0	-736.0
3	-1322.5	-198.4	300.0	-1220.9	-736.0	-147.2	320.0	-563.2
4	-1220.9	-183.1	600.0	-804.0	-563.2	-112.6	320.0	-355.8
5	-804.0	-120.6	900.0	-24.6	-355.8	-71.2	320.0	-107.0
Normal NCF - Project A - at 14.67% (IRR)				Normal NCF - Project B - at 18.03%(IRR)				
0			-1000.0				-1000	
1	-1000.0	-146.7	0.0	-1146.7	-1000.0	-180.3	320.0	-860.3
2	-1146.7	-168.2	0.0	-1314.9	-860.3	-155.1	320.0	-695.4
3	-1314.9	-192.9	300.0	-1207.7	-695.4	-125.4	320.0	-500.8
4	-1207.7	-177.1	600.0	-784.9	-500.8	-90.3	320.0	-271.1
5	-784.9	-115.1	900.0	0.0	-271.1	-48.9	320.0	0.0
Non-Normal NCF - Project C - at 5.0% Interest				Non-Normal NCF - Project D - at 10.0% Interest				
0			-1000.0				-10000.0	
1	-1000.0	-50.0	800.0	-250.0	-10000.0	-1000.0	3000.0	-8000.0
2	-250.0	-12.5	1000.0	737.5	-8000.0	-800.0	4000.0	-4800.0
3	737.5	36.9	1300.0	2074.4	-4800.0	-480.0	-1000.0	-6280.0
4	2074.4	103.7	-2200.0	-21.9	-6280.0	-628.0	8000.0	1092.0
Non-Normal NCF - Project C - at 10% Interest				Non-Normal NCF - Project D - at 15.0% Interest				
0			-1000.0				-10000.0	
1	-1000.0	-100.0	800.0	-300.0	-10000.0	-1500.0	3000.0	-8500.0
2	-300.0	-30.0	1000.0	670.0	-8500.0	-1275.0	4000.0	-5775.0
3	670.0	67.0	1300.0	2037.0	-5775.0	-866.3	-1000.0	-7641.3
4	2037.0	203.7	-2200.0	40.7	-7641.3	-1146.2	8000.0	-787.4
Non-Normal NCF - Project C - at 6.6% (IRR)				Non-Normal NCF - Project D - at 13.0% (IRR)				
0			-1000.0				-10000	
1	-1000.0	-66.0	800.0	-266.0	-10000.0	-1300.4	3000.0	-8300.4
2	-266.0	-17.6	1000.0	716.4	-8300.4	-1079.4	4000.0	-5379.8
3	716.4	47.3	1300.0	2063.7	-5379.8	-699.6	-1000.0	-7079.4
4	2063.7	136.2	-2200.0	0.0	-7079.4	-920.6	8000.0	0.0

In Table 1, Project A and B used data from Silber NPV vs IRR⁶ and Project C used data from Damodaran⁷ and project D is a hypothetical case.

Table: 2. Comparison of NPV and IRR Estimated with CAS and DCF Methods

Estimate Details	Normal NCF		Non-Normal NCF	
	Project A	Project B	Project C	Project D
I. Estimates Based on CAS Method				
a. Final closing balance (CB) at 10%	312.5	343.1	40.7	1092.0
b. Present Value of CB at 10% (= NPV)	194.0	213.1	27.8	745.9
c. NPV at 10% by the new method	194.0	213.1	27.8	745.9
d. IRR by the new method (CB = 0)	14.67%	18.03%	36.55% (6.6%)*	13.0%
e. IRR estimated by interpolation	14.67%	18.03%	36.55% (6.6%)*	13.0%
f. IRR without reinvestment income	14.67%	18.03%	-9.0% ⁺	13.0%
f. Final closing balance (CB) at IRR	0.0	0.0	0.0	0.0
g. Present Value of CB at IRR (= NPV)	0.0	0.0	0.0	0.0
II. Estimates Based on DCF Method (Excel)				
a. Estimated NPV at 10%	194.0	213.1	27.8	745.9
b. Estimated IRR	14.67%	18.03%	36.5% (6.6%)*	13.0%
c. Estimated NPV at IRR	0	0	1	1

* Project C, with NNCF, ends up with multiple IRRs 36.55% and 6.6% (in parenthesis). IRR of 6.6% is estimated by interpolation of NPVs at 5% (PV of -\$21.7 = -\$18.01) and NPV at 10% (\$27.8).

+ Unique IRR after elimination of reinvestment income and the NPV is zero at -9.0%.

b. The important finding is that the PV of the CB in a CAS at a particular interest rate is the NPV at that interest rate applied (Eq. 5 and 6). The rate that makes the CB zero (also NPV = 0) is the IRR (Eq. 7). The estimated IRR and NPVs by CAS method is consistent with the DCF estimated NPV and IRR.

c. Alternatively, the IRR can be estimated by interpolation using the estimated NPVs under the CAS method. A higher and a closest lower discount rate are used in the CAS to estimate the CBs at those rates. The PV of negative and positive CBs at two different rates (higher and lower rates) are the NPVs at those interest rates. Those interest rates and the NPVs at those rates are used to estimate the IRR by interpolation and compared. The results are presented in Table 1.

d. CASs at IRR, as interest rate, leads to CBs of '0' in all the four cases (NCF and NNCF) studied. The NPVs and the PV of the CBs, are therefore '0' at IRR. These results indicate that the NCFs are fully utilized to pay of the ROC and highest possible ROIC equal to IRR. It can be also inferred

⁶ Source: NPV Versus IRR, W.L. Silber, project details available in website <http://people.stern.nyu.edu/wsilber/NPV%20Versus%20IRR.pdf>

⁷ Source Damodaran: "Multiple IRR Project Cash flow", www.stern.nyu.edu/~adamodar%20Fpc%20Fcf2Eill%20Fip10p12.xls

that the NCFs support a maximum ROIC equal to IRR and the recovery of the ROC in full. The NPV is zero at IRR which is consistent with the mathematical relationship between IRR and NPV as expected.

4.2 Comparison of the DCF vs CAS method using large projects funded by the ADB:

The suitability of the CAS method for large scale projects with 25 or more years of life and for the financial and economic analysis of projects is evaluated next. Two large projects, one each from Vietnam and Laos, funded and appraised by the ADB consultants are selected for comparison. The NCFs of investment projects, irrespective of small or large investments, are the input data used in DCF method so also in the CAS method to estimate the IRR and the NPV. In financial analysis the IRR and NPV are referred as FIRR and FNPV and in economic analysis as EIRR and ENPV. The financial NCF (FNCF) will be converted into economic NCF (ENCF) by using standard conversion factors, shadow prices and by eliminating transfer payments⁸. The DCF and CAS methods use FNCF or ENCF to estimate FIRR or EIRR. The estimated FIRR or EIRR under both methods are compared to illustrate that the results are consistent.

The results are presented in Table 3 and 4 and the findings are discussed here. In table 3 and 4, the first year OB is zero and the first year (2013) NCF is shown as the OB of second year (2014). This assumes end-of-year (EOY) cashflow i.e., all the cashflows accrue or accounted for at the end of the year only. The CF is NCF for the respective years.

a. In the case of Vietnam project (table 3), the ADB consultant used the project NCF and estimated the NPV at 12% \$1046105677 (see foot note 4 for the correction) and IRR of 24.5%. The same NCF data is used in CAS. The CAS at 12% rate leads to a closing balance of \$177838883. The PV of the CB at 12% is \$1046105677 which is the NPV. Similarly, the CAS at 24.5% using the same NCF ends up with a closing balance '0' and thereby indicating that 24.5% is the IRR.

b. In the case of Laos project (table 4), the ADB consultants estimated only IRR (FIRR and EIRR) and not the NPV. Conventionally, NCF for 25 to 30 years will be sufficient because after 30 years the responses to discount rates are almost negligible (flat curve). For comparison, the life of project is considered as 45 years.

c. The same FNCF and ENCF used in DCF method by the ADB consultants are used in the CAS method. The estimated FIRR of 7.63% and EIRR of 11.7% using CAS method perfectly match with the DCF method estimates.

In brief, whether it is small or large project, and the estimates are financial (FIRR) or economic (EIRR), the estimated IRR and NPV by the CAS method match with the DCF estimates.

⁸ For explanations, please refer to:

<https://web.stanford.edu/group/FRI/indonesia/documents/gittinger/Output/chap7.html>

Table: 3. Comparison of the DCF vs CAS method estimated IRR and NPV with a Real-Life Project Feasibility Study on GMS Ha Noi to Lang Son Expressway: ADB: Project No.: TA No.7154 - Vietnam Financial Viability (Amount US\$ million)

Year	NCF	PV of CB at 12% = NPV				CB at 24.5% = 0 (i.e. IRR = 24.5%)			
		OB	ROIC	CF	CB	OB	ROIC	CF	CB
2013	-82094550	0	0	-82094550	-82094550	0	0	-82094550	-82094550
2014	-190211015	-82094550	-9851346	-190211015	-282156911	-82094550	-20111017	-190211015	-292416582
2015	-178699317	-282156911	-33858829	-178699317	-494715057	-292416582.3	-71634413	-178699317	-542750313
2016	-209463239	-494715057	-59365807	-209463239	-763544103	-542750312.7	-132959629	-209463239	-885173181
2017	-30305003	-763544103	-91625292	-30305003	-885474399	-885173180.6	-216844274	-30305003	-1132322458
2018	61581618	-885474399	-106256928	61581618	-930149708	-1132322458	-277389382	61581618	-1348130222
2019	168443819	-930149708	-111617965	168443819	-873323854	-1348130222	-330256639	168443819	-1509943042
2020	242700977	-873323854	-104798863	242700977	-735421740	-1509943042	-369896547	242700977	-1637138612
2021	279024609	-735421740	-88250609	279024609	-544647740	-1637138612	-401056134	279024609	-1759170137
2022	281585301	-544647740	-65357729	281585301	-328420168	-1759170137	-430950666	281585301	-1908535502
2023	395800823	-328420168	-39410420	395800823	27970235	-1908535502	-467541273	395800823	-1980275952
2024	430779244	27970235	3356428	430779244	462105908	-1980275952	-485115807	430779244	-2034612515
2025	444235324	462105908	55452709	444235324	961793941	-2034612515	-498426843	444235324	-2088804034
2026	473592828	961793941	115415273	473592828	1550802041	-2088804034	-511702348	473592828	-2126913554
2027	535514218	1550802041	186096245	535514218	2272412504	-2126913554	-521038183	535514218	-2112437520
2028	529844615	2272412504	272689501	529844615	3074946620	-2112437520	-517491934	529844615	-2100084838
2029	495617374	3074946620	368993594	495617374	3939557588	-2100084838	-514465850	495617374	-2118933314
2030	565858179	3939557588	472746911	565858179	4978162678	-2118933314	-519083233	565858179	-2072158368
2031	558412479	4978162678	597379521	558412479	6133954678	-2072158368	-507624595	558412479	-2021370485
2032	601128633	6133954678	736074561	601128633	7471157873	-2021370485	-495182892	601128633	-1915424744
2033	599254750	7471157873	896538945	599254750	8966951567	-1915424744	-469228957	599254750	-1785398951
2034	640029515	8966951567	1076034188	640029515	10683015270	-1785398951	-437376039	640029515	-1582745475
2035	674226137	10683015270	1281961832	674226137	12639203240	-1582745475	-387731239	674226137	-1296250577
2036	639246020	12639203240	1516704389	639246020	14795153649	-1296250577	-317547483	639246020	-974552040
2037	1213291797	14795153649	1775418438	1213291797	17783863883	-974552040.2	-238739757	1213291797	0.00

Source: <https://www.adb.org/sites/default/files/project-document/61285/41414-01-vie-tacr-01.pdf> Table: 8.36. Financial Viability (NCF is financial value and therefore the IRR is referred as Financial IRR (FIRR))

Estimate as per Project report by DCF Method	Estimate by CAS method
IRR (FIRR) = 24.50%	IRR (FIRR) = 24.50% that made the CB = 0
NPV at 12% = 1046105677 ⁹ ; PV of CB \$ 17783863883 = 1046105677 (NPV at 12%)	CB at 24.50% = 0, PV of CB = NPV at 24.5% = 0

⁹ Note: NPV as per Project report is 934,022,925 based on NCF of 26 years but the actual NCF is 25 years.

Table: 4. Comparison of the DCF vs CAS method estimated IRR with a Real-Life Project

ADB: Project Performance Audit Report: PPA: Lao 29163 -Nam Leuk Hydropower Project – Lao

Table A6.1: Financial IRR (FIRR) - CAS at 7.63%					Table A6.2: Economic IRR (EIRR) - CAS at 11.77%				
Year	OB	ROIC	CF (NCF)	CB	OB	ROIC	CF (NCF)	CB	
1997	0	0	-12738	-12738	0	0	-12672	-12672	
1998	-12738	-972	-25914	-39624	-12672	-1491	-25781	-39944	
1999	-39624	-3024	-33756	-76404	-39944	-4701	-33618	-78263	
2000	-76404	-5831	-11657	-93892	-78263	-9210	-11534	-99007	
2001	-93892	-7166	2877	-98181	-99007	-11651	2923	-107735	
2002	-98181	-7493	6051	-99624	-107735	-12678	6088	-114325	
2003	-99624	-7603	6390	-100837	-114325	-13454	6425	-121354	
2004	-100837	-7696	6426	-102107	-121354	-14281	6461	-129174	
2005	-102107	-7793	6731	-103169	-129174	-15201	6766	-137609	
2006	-103169	-7874	7960	-103082	-137609	-16194	11661	-142142	
2007	-103082	-7867	9009	-101941	-142142	-16727	17095	-141775	
2008	-101941	-7780	8991	-100730	-141775	-16684	17137	-141322	
2009	-100730	-7688	8963	-99454	-141322	-16631	17201	-140752	
2010	-99454	-7590	8428	-98617	-140752	-16564	17006	-140309	
2011	-98617	-7526	8577	-97566	-140309	-16512	17049	-139772	
2012	-97566	-7446	8610	-96403	-139772	-16448	17112	-139108	
2013	-96403	-7357	8632	-95128	-139108	-16370	17154	-138325	
2014	-95128	-7260	8367	-94021	-138325	-16278	16650	-137953	
2015	-94021	-7176	8313	-92884	-137953	-16234	17030	-137157	
2016	-92884	-7089	8345	-91628	-137157	-16141	17092	-136206	
2017	-91628	-6993	8366	-90255	-136206	-16029	17134	-135101	
2018	-90255	-6888	8398	-88745	-135101	-15899	17196	-133803	
2019	-88745	-6773	8419	-87099	-133803	-15746	17237	-132312	
2020	-87099	-6647	8451	-85295	-132312	-15571	17237	-130646	

Source: <https://www.adb.org/sites/default/files/project-document/69873/ppar-lao-29163.pdf>

Table: 4. Comparison of the DCF vs CAS method estimated IRR with a Real-Life Project (Continue)

ADB: Project Performance Audit Report: PPA: Lao 29163 -Nam Leuk Hydropower Project – Lao

Table A6.1: Financial IRR (FIRR) - CAS at 7.63%					Table A6.2: Economic IRR (EIRR) - CAS at 11.77%				
Year	OB	ROIC	CF (NCF)	CB	OB	ROIC	CF (NCF)	CB	
2021	-85295	-6510	8451	-83354	-130646	-15374	17237	-128783	
2022	-83354	-6362	8451	-81265	-128783	-15155	17237	-126702	
2023	-81265	-6202	8451	-79016	-126702	-14910	17237	-124375	
2024	-79016	-6031	8451	-76596	-124375	-14636	17237	-121774	
2025	-76596	-5846	8451	-73990	-121774	-14330	17237	-118868	
2026	-73990	-5647	8451	-71186	-118868	-13988	17237	-115619	
2027	-71186	-5433	8451	-68168	-115619	-13606	17237	-111988	
2028	-68168	-5203	8451	-64920	-111988	-13179	17237	-107930	
2029	-64920	-4955	8451	-61423	-107930	-12701	17237	-103394	
2030	-61423	-4688	8451	-57660	-103394	-12167	17237	-98325	
2031	-57660	-4401	8451	-53610	-98325	-11571	17237	-92659	
2032	-53610	-4092	8451	-49250	-92659	-10904	17237	-86326	
2033	-49250	-3759	8451	-44558	-86326	-10159	17237	-79248	
2034	-44558	-3401	8451	-39508	-79248	-9326	17237	-71336	
2035	-39508	-3015	8451	-34072	-71336	-8395	17237	-62494	
2036	-34072	-2600	8451	-28222	-62494	-7354	17237	-52612	
2037	-28222	-2154	8451	-21924	-52612	-6191	17237	-41566	
2038	-21924	-1673	8451	-15147	-41566	-4891	17237	-29220	
2039	-15147	-1156	8451	-7852	-29220	-3439	17237	-15422	
2040	-7852	-599	8451	0	-15422	-1815	17237	0	

Source: <https://www.adb.org/sites/default/files/project-document/69873/ppar-lao-29163.pdf>

Estimate as per Project report by DCF Method	Estimate by CAS method
IRR (FIRR) = 7.63%	IRR (FIRR) = 7.63% that made the CB = 0
IRR (EIRR) = 11.77%	IRR (EIRR) = 11.77% that made the CB = 0

4.3 Transparency of CAS method:

The reinvestment assumption and the problem of multiple IRR are some of the major controversies relating to CBA and CIA. Economists continues to debate this issue for more than a half century. The CAS method transparently reveals if there is any reinvestment of intermediate income and how the reinvestment can lead to multiple IRR and how multiple IRR could be eliminated. CAS estimates also provide evidence to understand the appropriate criterion (IRR vs NPV).

Reinvestment of intermediate income: A careful analysis of the CAS estimates reveal whether there is any reinvestment of intermediate income as discussed here (see also Arjunan and Kannapiran, 2017):

- In Table 1, projects A and B are with normal NCF, and they pay-off ROIC as indicated by the negative interest (interest expenses or interest paid by the project). No positive interest income flows into the project NCF and therefore no reinvestment of intermediate income.
- Although both projects C and D (Table 1) are NNCF projects, only project C receives interest income during years 3 and 4 as revealed by the positive interest income. The positive income is due to the positive opening balance (OB) in years 3 and 4 due to the nature of the NNCF. The positive interest income (income from reinvestment) is generated by the reinvestment of intermediate income.
- The NNCF project D does not end up with credit balance during any of the four years of the project life due to the nature of its NNCF. No interest income flows into the project NCF as could be seen in table 1 and therefore there is no reinvestment of intermediate income. This is an important finding that reinvestment is not a problem with all NCF but with some (not all) NNCF. Identifying the NCCF that will lead to reinvestment and multiple IRR is an important area for future research.

The inferences include a. Normal NCF investment does not involve reinvestment of intermediate income; b. Not all NNCF projects involve reinvestment; c. Only NNCF projects with intermediate positive OB in some of the years of the project life are the cases of reinvestment of intermediate income (see table 1). The CAS transparently reveals whether there is any reinvestment or not. In summary, the CAS method indicates that there is no reinvestment of intermediate income in the case of normal NCF but there is reinvestment with some of the NNCF projects.

Multiple IRR: Normal NCF investments always end up with a unique IRR (project A and B). Among NNCF projects (C and D), only projects with positive intermediate OB (and reinvestments) ends up with multiple IRR (project C). NNCF project with no positive OB in any years leads to unique IRR (project D). When the reinvestment income during years 3 and 4 are excluded from the NCF for the project C, the problem of multiple IRR is eliminated. Accordingly, the unique IRR of -9.0% for project C seems realistic when the sum of NNCF is -100 (about 10% capital loss: 100/1000). This is an interesting finding that resolves the century old problem of multiple IRRs (details available in Arjunan, 2017a).

One of the important findings is that the NNCF projects with reinvestment of intermediate income end up with multiple IRR. When the reinvestment income is excluded from the NCF, the estimated IRR is unique even for the NNCF investments. The twin problems of reinvestment at IRR and multiple IRR are resolved.

NPV vs IRR as a Criterion: The debate on the appropriate criterion (IRR vs NPV) has been continuing for long. As the CAS method provide a better insight into these criteria (IRR and NPV), an attempt is made here to present some numerical analytical evidence.

CASs for four projects, including one non-normal NCF, are prepared for various interest or discount rates and presented in Table: 5. As could be seen from Table 5, the summation of NCFs (CF_1 to CF_t) over the life of the investment perfectly matches with the sum of ROC, ROIC in \$ or in absolute term and the CB for the life of the investment. The estimates reveal how the NCF is allocated to ROC, ROIC and CB for various projects. Higher the interest rate, lower will be the CB so also the NPV. At interest rate equivalent to IRR, the CB becomes zero ($NPV = 0$). The interest or discount rates play a pivotal role in the allocation of the NCF (as in loan amortization). The results of numerical analysis are presented in Table 5 and discussed below.

a. First, cumulative $NCF = ROC + ROIC$ in \$ terms + CB as presented for various interest rates for four investment projects in Table 5. ROC is the initial investment that remains unchanged (unless there is some capital replacement cost and in such a case the NCF becomes non-normal). The ROIC is the cumulative interest amount paid (in \$) during the life of the project and if that vary, the variable CB will also vary.

b. At 10% interest rate, the CB is positive and therefore the PV of CB (= positive NPV) is also positive which is considered as the value added after ROC and cost of capital (say 10%). Similarly, at interest rates below the IRR the CB is positive (positive NPV). The positive NPV reflects the PV of the unutilized NCF.

c. When the interest rate is higher than IRR, the CB becomes negative and therefore the NPV is negative. The ROIC is higher than what could be supported by the NCF and therefore CB remains negative.

d. With IRR as the interest rate, the CB is zero and therefore PV of CB is also zero ($NPV = 0$). Here the sum of ROC plus ROIC (in absolute term) perfectly match with the NCF and therefore the NCF is fully utilized.

As discussed above, CAS method is more transparent and provides a better insight into what constitutes NPV. It also indicates that the IRR fully utilises the NCF and ensure full recovery of ROC and the highest possible ROIC. With this background, the appropriateness of NPV and IRR as a selection and ranking criterion is further evaluated and discussed below.

a. At interest rates $< IRR$, end up with positive CBs in all cases. The PVs of these positive CBs result in positive NPVs and therefore NPV represents the unutilized or under-utilized NCFs. The NCFs are not fully utilized to achieve the highest possible ROIC (i.e., IRR). The unutilized NCFs remain as the CBs in all cases and the positive NPV is an indicator of the unutilized portion of the NCF. NPV failed to indicate the potential ROIC in percentage terms. It only reflects that the NCF is still available to increase the ROIC. NPV therefore has limited use in ranking or selecting projects.

Table: 5. CAS and allocation of the NCF to ROC, ROIC and CB for various projects

Net Cash flow (NCF)		Interest / Discount Rate	Return of capital (ROC)	Cumulative Interest on capital (ROIC in amount)	Closing Balance (CB from CAS)	Cumulative Net Cash inflow = (ROC + ROIC + CB)
Year	Amount in \$	1. Data Source: Rich, S.P. and Rose, J.T. (2014): \$				
0	-77000					
1	35000	10.00%	77000	14637	13363	105000
2	35000	15.00%	77000	23570	4430	105000
3	35000	17.3% ¹	77000	28000	0	105000
		20.00%	77000	33656	-5656	105000
Year	Amount in \$	2. Data Source: Osborne, M.J. (2010): Project B- \$ million				
0	-100					
1	50	10.00%	100	15.4	44.6	160
2	50	15.00%	100	41	19	160
3	40	25.43% ¹	100	60	0	160
4	20	30.00%	100	79.3	-19.26	160
Year	Amount in \$	3. Data Source: Silber, W.L. (NPV vs IRR, 2016): Project B: \$				
0	-1000					
1	320	10.00%	1000	257	343	1600
2	320	15.00%	1000	454	146	1600
3	320	18.0% ¹	1000	600	0	1600
4	320	25.00%	1000	1026	-426	1600
5	320					
Year	Amount in \$	4. Non-normal NCF – Data Source Damodaran (2010)				
0	-1000					
1	800	10.00%	1000	-140.7	40.7	900
2	1000	30.00%	1000	-54.6	-45.4	900
3	1300	36.5% ¹	1000	-100	0	900
4	-2200	40.00%	1000	-33.6	-66.4	900

Note: Super script '1' indicates IRR.

b. These findings demonstrate that the IRR truly reflects the full potential of the NCF from the investment (to recover ROC and ROIC) whereas NPV, being a point estimate, only indicates the cut-off point to accept or reject. NPV fails to indicate the maximum ROIC possible for a given NCF. NPV does not indicate the value added to the firm as argued by many authors or the economic return¹⁰ (must be based on economic NCF) as strangely claimed by Tang and Tang (2003).

¹⁰ Economic rate of return is discussed in detail in section 4.4.

c. When NPV and IRR differ in ranking mutually exclusive investments, the conventional suggestion is to consider NPV as the better criterion. This convention must be reviewed because NPV reveals the ROIC in part in percentage term (hurdle rate or cost of capital) and the unutilized ROIC in absolute terms (\$) whereas IRR reveals in percentage terms the full ROIC. Only when both criteria are in same term a better comparison could be made. The objective function of any corporate management is to maximize the return or the profitability of the investment. Obviously, corporate managements consider that the NPV at hurdle rate (10%) as the bottom line and a higher return is always the target. Higher positive NPV reveals that there is more unutilized NCF (ROIC in absolute term) and therefore the ROIC could be maximized equivalent to IRR. These are the main reasons for the private sector to prefer IRR than NPV (see Osborne, 2010; Brealey et al. 2009; Bhattacharyya, 2004). NPV is a restricted estimate and provides incomplete information to support investment or to select or rank mutually exclusive investment projects.

d. An incidental inference is that the MIRR is redundant when there is no reinvestment of intermediate income with normal NCF investments. In the case of NNCF that has positive OB in any years, the reinvestment income can be deleted under the new method and here also the MIRR is irrelevant (see Arjunan, 2017b).

5. Summary and conclusions:

A new method to estimate the NPV and IRR is presented which is based on CAS. The CAS method estimated NPV, and IRR perfectly match with the NPV, and IRR estimated by the DCF method. The new method is more transparent and enables better understanding of the problems of reinvestment, multiple IRR and the suitability of IRR and NPV in CBA and CIA. The results are summarized here.

- a) A set of functional equations used in DCF method to estimate NPV and IRR is simplified. The simplified equation enabled to identify that the NPV is the PV of the closing balance in CAS at a particular rate and the rate that makes closing balance zero is the IRR. The identified numerical relationship is used to develop and present the CAS method to estimate the NPV and the IRR.
- b) Data from selected investment projects (normal and non-normal NCF), both from academic publications and real-life projects, are used to prepare CASs and to estimate the closing balance for each CAS. The PV of the closing balance is estimated as NPV at a specific rate. The interest rate that makes the CB zero ($NPV = 0$) is the IRR. The NPVs estimated by CAS method at various interest or discount rates are used to estimate IRR by interpolation method also.
- c) The estimated NPV and IRR using the CAS data are compared with the NPV and IRR estimated by DCF method. The results perfectly match and therefore the CAS method is very useful.
- d) The new method of estimation of NPV and IRR transparently reveals that:
 - there is no reinvestment of intermediate income in the case of normal NCF investments and not all the NNCF involve reinvestment of intermediate income;
 - NNCF with positive CB in some years leads to reinvestment of intermediate income;
 - the controversial assumption of reinvestment is also rejected in normal NCF and some of the NNCF investments; These investments do not lead to the problem of multiple IRR.

- reinvestment income should not be considered as project benefit. By excluding the reinvestment income from the NCF, the problem of multiple IRR is resolved;
- the MIRR, which is based on reinvestment assumption, becomes redundant.

e) The CAS method also clearly indicates that the NCF is the sum of ROC, ROIC and the CB. With IRR (the highest ROIC), the closing balance becomes zero and therefore the PV of CB, that is NPV, is zero. The NPV is the unutilized NCF, being the PV of the CB, but the investors are interested to know the maximum return that the NCF could generate. IRR estimate fully utilize the NCF and therefore provides a better indicator that the investors are searching for. NPV is, therefore, not a valuable criterion in CBA and CIA.

Conclusion

The CAS method to estimate the NPV and IRR using NCF data is presented in this paper. The PV of the CB, derived from the CAS, is the NPV and the rate that makes the CB zero is the IRR. The results of the estimated NPV and IRR using the new method perfectly match with the prevailing DCF method estimates in excel.

a. The CAS method reveals that the controversial assumption of reinvestment is not valid as the CAS does not include any reinvestment income in the cases of normal NCF and in some cases of NNCF.

b. There is no multiple IRR problem when there is no reinvestment. When there is reinvestment in some NNCF investments, the multiple IRR problem can be resolved by deleting the reinvestment income from the benefit stream in CAS method.

c. As there is no reinvestment of intermediate income in all cases, as revealed by the CAS, the modified IRR (MIRR) is obviously not valid as MIRR is based on reinvestment assumption.

d. The new method reveals that IRR is a better criterion than the NPV to accept projects and rank mutually exclusive projects too. NPV is the unutilized NCF and a static point estimate (at hurdle rate) and an incomplete criterion.

These findings present a case to review the Corporate Finance Economic texts and CBA or Investment Analysis or Project Analysis guidelines by the respective authors and to include the new method, highlight the problems with NPV and discuss the usefulness of IRR as the best criterion for investment decisions. Future research needs to identify the NCF that will specifically lead to reinvestment and multiple IRR.

References

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